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THE UNIVERSITY OF ALBERTA

MATRICULATION STANDARDS OF ALGEBRA 2 STUDENTS AS DETERMINED BY THE 1948 MIDSUMMER EXAMINATION.

A DISSERTATION

SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF EDUCATION

FACULTY OF EDUCATION

BY

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FOREWORD

For the past 2000 years, Mathematics has been utilized in one way or another by people in various walks of life. The question has often been posed as to the necessity of Mathematics teaching. Is it necessary for teachers to burden the young with abstruse and abstract ideas, when in all probability the amount of Mathematics required by any one child will be strictly limited to a knowledge of the four fundamental processes? The argument then boils down to "Lets just teach sufficient Mathematics, so that the child can get along in society."

However, on deeper reflection we find that Mathematics plays a singularly important part in our every day living. Whether we like it or not, Mathematics is continually with us. No aspect of every day living is completely devoid of it.

If we accept the purpose of education in general as being that which enables each individual to adjust himself to those elements of his environment that are of concern to him in modern life, and to develop, organize and train his powers so that he

may make efficient and proper use of them, then Mathematics will play an important part in educating the pupil.

Clearly then, the aim of a Mathematics education should be:

(1) To give the pupils a mastery of those phases of Mathematics which are necessary for him in his social and material environment.

(2) To enable an individual to recognize a mathematical situation when he meets one.

(3) To give each individual the mathematics necessary for his vocation.

(4) To enable an individual to discover new and more efficient methods of procedure.

(5) To give gifted individuals who plan on specializing in Mathematics or related fields a mastery of the elements of Mathematics.

(6) To enable the student to use Mathematics as a tool in solving the problems of his environment.

To enable the right pupil to receive the right education from the right teachers may be considered the aim of a good educational system.

Presupposing the right teachers we may define a right pupil as one whose personal attributes and opportunities enable him to profit fully from the education offered him. The right education is the one which is best suited to the needs of society and the pupil. The most obvious way of determining the rightness of pupils and education is a task for the process of evaluation.

Evaluation then serves as a tool by means of which we are able to guide a pupil into a desired or chosen field. Because of individual differences which exist between students, it is obvious that not all of them can be guided into the Mathematics field.

Evaluation then, serves to:

(1) Maintain standards. Standards furnish the means of social control and evaluation determines whether or not the standards are satisfied.

(2) Select students. Selection prevents misfits from entering into vocations or studies for which they show no capabilities.

(3) Motivate learning and guide teaching.

(4) Appraise teachers, teaching methods, books and curricular content.

Thus if we are to teach Mathematics in our High Schools then we must evaluate.

CHAPTER I

PURPOSE OF THE INVESTIGATION

I The Examination as a Whole

In June 1945, 2853 Grade XII Students wrote an examination in Algebra 2. The examination was set on the material in the text "Senior Algebra" by Durell and Wright and edited by Dr. E. W. Sheldon and Mr. D. L. Shortliffe. The answer papers were marked and the ranking of the candidates and the division of the scores into groups was completed before this investigation was started.

The raw scores obtained by the 2853 candidates ranged from 13 to 147. The following table shows the number of candidates whose scores fell within certain intervals. (The interval is 9 raw scores.)

TABLE I

FREQUENCY OF SCORES OF 2853 CANDIDATES ON THE 1948 ALGEBRA 2 EXAMINATION

Raw Score Interval	Midpoint	Frequency	Cumulative Frequency
9-17	13	5	5
18-26	22	26	31
27-35	31	93	124
36-44	40	216	340
45-53	49	244	584
54-62	58	328	912
63-71	67	394	1306
72-80	76	364	1670
81-89	85	350	2020
90-98	94	306	2326
99-107	103	208	2534
108-116	112	151	2685
117-125	121	99	2784
126-134	130	45	2829
135-143	139	20	2849
144-152	148	4	2853

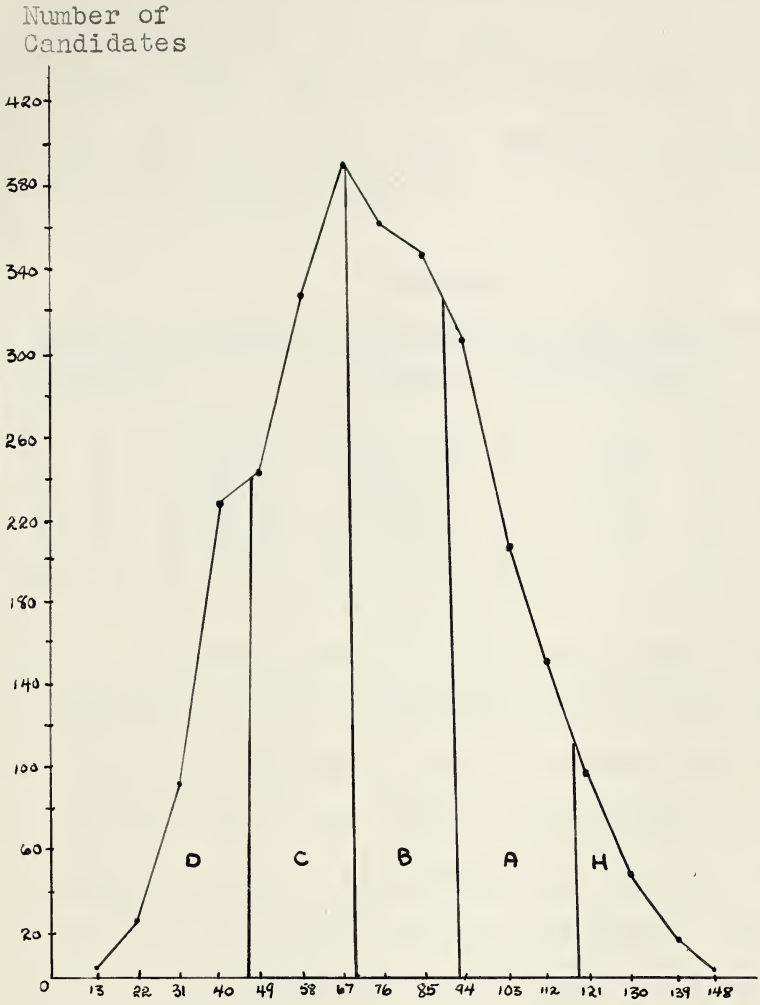


Fig. I

Fig. I is a graphical representation of the data contained in Table I.

The following table shows the ranking of the 2853 candidates and the range of scores for each ranked group. The vertical lines in Fig. I divide the graph into five parts. The data of this table are thus shown on Fig. I.

TABLE II

Letter Grading	% Range	Raw Score Range	Number of Candidates	%
H	80-100	119-147	148	5.2
A	65-79	93-118	572	20.1
B	50-64	68-92	969	34.6
C	40-49	48-67	731	25.6
D	0-39	0-47	413	14.5

An Examination of this table shows that 1144 candidates made a percentage mark of less than 50, (fewer than 68 raw scores), after the raw scores had been adjusted to the score range 0-100; while 1709 candidates made a percentage mark of more than 50 after the raw scores had been adjusted. Further, of the 1709 candidates 720 of them obtained a mark of 65 or better, while 989 candidates received a mark of between 50 and 64. This latter group comprised 35% of the total number of candidates.

II The Answer Papers

The 300 papers used in this investigation were secured from the Department of Education. These answer papers represent all the candidates whose raw scores were 68,69,70,71,72,73 and as many with a raw score of 74 as was required to bring the total number to 300. These raw scores corresponded to a percentage score of 50,51,51,52,52,53 and 54 respectively.

The following table shows the range of scores (raw and transmuted) for the 300 cases with which this investigation is concerned.

TABLE III

Raw Score	Transmuted Score	Frequency
68	50	45
69	51	49
70	51	43
71	52	39
72	52	51
73	53	39
74	54	34

III The Examination Paper

The examination paper consisted of 53 questions having a total value of 153 marks. It is significant to note that the highest raw score obtained by a candidate was 147. The first 34 questions were of the Multiple Choice Type and answers were recorded as marks in suitable spaces on the provided sheet. This section of the paper was machine scored. The second half of the paper consisted of questions of the Problem-solving type. This part of the paper was hand marked. Each question in the first part of the paper was valued at 2 raw scores. Questions on the second part of the paper were valued from 3 to 6 raw scores.

IV Purpose of the Investigation

The purpose of this investigation is to find out the matriculation standards as set by the pupils' scores on this examination. From the data of Table II it can be seen that any candidate who secured 68 raw scores received a percentage grading of 50. Thus it was not necessary to secure one half of the 153 raw scores in order to get 50%. However, it must be remembered that no student was able to secure a

perfect score.

Further problems which require attention in connection with the above named purpose are:

(1) What does a pupil just at the lowest matriculation level, know?

(2) How adequately has the examination covered the material of the text book?

(3) On the basis of the course, was the examination a fair test?

(4) On the basis of results for this group, was the division into groups as indicated by Table II a fair one?

CHAPTER II

DEFINITIONS AND PROCEDURE

As Previously stated, the 300 cases under consideration in this investigation comprise all the cases who received raw scores of 68 to 73. It was necessary in order to have exactly 300 cases to take 34 cases whose raw scores were 74. This "74 raw score group" contained some 44 cases. The group was not sampled. The cases were taken in alphabetical order of last names of candidates as shown on the Examination application forms at the Department of Education, Examinations Branch.

I Definitions

(a) Positive Analysis

The term "positive analysis" as used in this dissertation will mean an analysis of the answer papers for correct scores. The 300 papers were meticulously analyzed for the number of correct raw scores which a candidate was able to secure on any one question. In the case of the first 34 questtions this was a simple task. In considering answers to these questions no partial marks were given. The question was either wrong or received a perfect

score, in this case, 2.

The last 19 questions on the paper were of the "problem solving" type. Here the situation was entirely different. Partial scores were awarded for partial answers, and these were recorded.

(b) Negative Analysis

The term "negative analysis" as used in this dissertation will mean an analysis of the answer papers for errors. Here too, as will be noted under the chapter headed "Negative Analysis", great care was taken to insure that all errors were recorded.

The first 34 questions were of the "multiple choice type". The correct answer was to have been selected from five given possible answers, only one of which was correct. Thus in carrying out an analysis of these questions, it is impossible to state definitely why a candidate answered as he did. Guess work in the solution could have been rewarded equally as well as an insight into the problem. Hence in the chapter dealing with the negative analysis. Responses other than the correct ones are indicated as errors.

The last 19 questions on the paper were of

the "Problem Solving type" and work was shown. Thus it was a relatively simple matter to check each problem to discover where the error was made. It may be noted here, that each problem was checked through to the final result and all errors in solution were recorded.

(c) No Attempts

In this dissertation a "No Attempt" will mean that the candidate omitted completely the question under consideration, i.e. there are no pencil marks or pen marks whatsoever to show that even a beginning was attempted.

In certain of the problems, sketches were drawn or some attempt was made such as writing down the data of the problem in a different order to what it was in the problem. These are counted as attempts; the candidate must have done some thinking in regard to the problem.

(d) Relationships

The definition of the term "relationship" as used in this investigation can best be expressed by an example. Question number 14 which is a question on differentiation- $\frac{d}{dx}(2x-4)(x+3)$ requires a knowledge,

on the pupils part, of the implication of the term $\frac{d}{dx}$ - the differential coefficient - in the solution of this problem. Thus a relationship involves a broad knowledge of the field of differentiation as in this example and the ability of the student to relate this problem to the general formula for differentiation of a function.

(e) Concepts

A "Concept" as used in this dissertation will mean the vocabulary of the course which the student gains as a result of instruction. The vocabulary comprises all those technical terms which he meets for the first time in Algebra 2. Examples of concepts are the words "maximum", "undefined" etc.

(f) Routines or Techniques

These terms will have a synonymous meaning in this investigation and they are defined as the ability, for example, to put a formula to work in order to arrive at an answer. They may aptly be termed the tools of the course.

II Procedure

The 300 papers used in this investigation are to be analyzed both positively and negatively, and a chapter of the dissertation will be given over to

each analysis.

The text book will be analyzed and all the concepts, relationships and routines or techniques will be listed. The examination paper, too will be anyalyzed for concepts, relationships and routines and the number of marks assigned to each will be stated. The score given for each will be determined from the scoring key which was used in marking the papers.

A chapter of the thesis will be given over to a comparison of text book coverage and examination coverage. This will determine just how much of the course the examination has covered. From these data, then, it will be possible to arrive at a statment of just how many raw scores the candidates were able to secure for each generalization concept and routine.

A comparison, will be attempted, with the results of examinations in Algebra 2 in the years 1935; 1938; 1940; 1941; 1942 and 1944, both as to correct scores and errors. The questions considered on the examination papers of the years stated abov

have been carefully selected and every effort has been made to select only those questions which are the same or nearly the same as those questions which appeared on the 1948 paper. It must be noted here that the cases selected for study for the years 1935; 1938; 1940; 1941 and 1942 are totally different from the cases which were selected for study from the 1948 paper. The closest approach to those under consideration in this thesis are the cases selected by Miss Miller in 1944. She selected 100 cases, which were all the students just above the 50% mark and including 50%. Then too the examination papers of 1935; 1938; 1940 and 1941 were totally different from the examination papers of 1942; 1944 and 1948. The latter three papers are very much the same in that the first 25 to 30 questions of each paper are of the short answer type while the remaining questions on the paper involve problems which require a written solution. The chapter dealing with Previous Related Studies will give an indication of just how the sample cases considered were selected.

A chapter of the Thesis will deal with just what percent of a topic the pupil knew on the basis

of his results for certain questions. The course has been divided into 15 topics and questions dealing with each topic have been lumped to-gether. Finally by use of a weighted mean method, the number of raw scores obtained on the topic, can be arrived at and this in turn can be expressed as a percent.

CHAPTER III

REVIEW OF PREVIOUS INVESTIGATIONS

I Summaries of Investigations in the Mathematical Field

Dr. Lazerte

This Thesis by Dr. M. E. Lazerte is a summary of investigations which were carried on in the Mathematics field in Alberta. One of these investigations concerned itself with the Grade XII Algebra examination of June 1935.

A random sample of 600 cases was considered. There were 200 cases from small country high schools, 200 cases from larger country and town high schools and 200 cases from city High Schools. The total number of candidates writing was 1880. An Analysis of correct scores and an error analysis was made of 400 cases. An Error analysis was not made for the 200 students who attended town schools.

1. Purpose of the investigation.

(1) To ascertain where the pupils had difficulty in the new course and the types of errors made. (This course in Algebra had been offered for two years only.)

(2) To compare the achievement of country school pupils with that of city school pupils.

2. Results

(A) There appeared to be outstanding deficiencies in:

(a) Graphing and sketching.

(b) Differentiation and meaning and use of derivatives.

(c) The limiting process.

(d) The integration process.

(e) Permutations and Combinations.

(f) Expansion by the Power Series.

(g) Summation of G. P.'s.

Many errors were also made in spelling, the manipulation of decimals and fractions and the use of logs.

(B) The students educated in small places showed consistently better progress and understanding than did students educated in the city - both in total marks on the paper and in success on individual questions. For example city pupils excelled the country pupils in only 4 parts of questions out of a total of 18 parts.

II A Report on the Algebra 3 Papers of June 1938
S. W. Hooper

Mr Hooper reviewed 329 papers, being a sample consisting of every eighth paper, ranging in marks from 0 - 100. In his initial remarks, Mr. Hooper concurs in the statement that the results showed, " a lamentable and widespread ignorance of the most elementary ideas and manipulation in the course."

The analysis of the papers dealt mainly with securing the number of correct answers and errors for the sample under consideration. Each question too was analyzed for the factors involved in a solution of the problem. A chapter of the dissertation is given over to a Summary of Errors.

Following are the conclusions which were arrived at:

(a) The paper appears well distributed over the years work.

(b) The examination paper was too difficult.

(c) The examination paper was too long.

(d) The form of the paper - one question per topic - may be an unfair examination for this type of course.

(e) The students' background may be such that the

proper teaching of Algebra 3 is an impossibility.

(f) Past examinations have not encouraged high standards.

III A Report on the Algebra 2 Papers of 1940-1941-1942
H. E. Miller

The purpose of this investigation was to analyze the errors made by the students and to discover, if possible, the cause for the consistent low scores in these three examinations.

No mention is made of how the papers were selected nor in what raw score or percentage range the papers fell.

The papers were analyzed for:

(a) Correct Solutions.

(b) Did not attempt problem.

(c) Common errors.

(d) Miscellaneous errors.

The following table shows the number of papers which were analyzed for each year:

1940 - 297

1941 - 309

1942 - 322

In summarizing results, Mr. Miller comes to the following conclusions:

(1) The examinations were excellent in allotment of marks for each section of the text.

(2) The problems on the examination seem to be fair samples of those in the text in regard to difficulty.

(3) Some problems were relatively easy, and some problems were relatively difficult.

The following table shows average results for different sections of the text book:

TABLE IV

% of Correct Responses and No Attempts on any one Topic - 1940 - 1941 - 1942.

Section of the Text	Pages in Text	% Correct	% did no Attempt Problem
Ratio, Proportion and Variation	22	36	19
Functions in one variable	20	37	28
Limits and Gradients	17	38	40
Differentiation	23	41	22
Integration	10	40	23
Sequence and Series	22	28	36
Permutations and combinations	11	31	14
Binomial Theorem	10	33	40
Empirical Formulae	9	20	53

A section of the dissertation is given over to special weaknesses as shown by the results. Mr. Miller classifies errors as being of three types which he lists as Class A, B, or C errors. He defines them thus:

(1) Class A errors - These reflect the prior training of the student as well as current training e.g. Arithmetic operations.

(2) Class B errors - Special Points of difficulty in the course e.g. understanding technical terms etc.

(3) Class C errors - Topics which seem to offer difficulty.

The following recommendations were made by the author:

1. The most outstanding weakness is in simple arithmetic. Therefore the following remedies should be practised:

   a. Insisting on a higher standard in the lower grades.

   b. Short practise periods in fundamental operations in the High Schools.

   c. Students should be taught to check results.

2. Teachers and examiners in marking papers should not allow nearly full marks for so-called mechanical errors.

3. Students should be taught to use the graphs which they draw.

4. The section on Annuities should be omitted or else treated more fully.

IV Student Attainment in the Algebra 2 Examinations of 1944 - V. I. Miller

For this investigation, Miss Miller selected 200 cases. One hundred cases were the students whose percentage mark was 39 and immediately above 39. The other one hundred cases were the students whose percentage mark was 50 and immediately above 50. These were the students who were just able to obtain a "C" and "B" grading respectively.

In the positive analysis of the paper it was found that "B" students consistently did better on all test items than "C" students with the exception of item 1 where "C" students excelled "B" students.

In the negative analysis "B" students showed consistent superiority over "C" students. "However it must also be admitted that even "B" students did

not have an acceptable background for university Mathematics and Science. They showed weakness in carrying out elementary Mathematical operations and in the application of the principles of analytical geometry to simple problems.

Further, it was found that the test items differed greatly in difficulty. Some items were found extremely easy by both groups. Others were found too difficult by both groups and still others were found to differentiate between the two groups very well.

In summing up her findings, Miss Miller states that perhaps the examination was a little too difficult. But taken as a whole, it was a valid test in the sense that it separated the students into two groups consistently.

The survey indicated a weak mathematical background for most students. It was recommended that perhaps teachers of mathematics might take more time to stress more of the fundamentals. More drill should be given on the fundamentals, and too, teachers should insist on greater accuracy. Further,

the short answer questions distinguished the clearer thinking "B" students from the muddled thinking of "C" students. Group "C" students also left more items incomplete. This group also omitted many more questions because they lacked insight into these problems.

CHAPTER IV

POSITIVE ANALYSIS

I Frequencies of Scores on the Various Test Items

The following frequency tables indicate how many students received a given mark on any one test item. Questions 1 to 34 were worth 2 raw scores each. No allowance was made for partial answers. These questions were objective in nature and the correct answer was to be selected from a group of 5 given answers.

Questions 35 to 53 were problem type questions. Partial marks were given for a partial solution. The correct raw scores on these questions varied from 3 to 6.

In order to give an indication of how the marks on any one question were allocated, the correct steps in the solution of question 35 to 53 are also given. The solutions indicated are those as set down by the Examinations Board. Some inconsistencies in marking were noted and these will be discussed in a later chapter.

The frequencies of correct scores and zero

scores are given as the number out of 300, indicated in the column headed N. The frequency is also expressed as a percent, as indicated under the column headed %. (Percentages are expressed to the nearest tenth.)

Questions, which were not attempted (as defined under procedure) were given a score of zero.

TABLE V

Frequencies of Scores on Questions 1 to 5.

Question	1		2		3		4		5	
Score	N	%	N	%	N	%	N	%	N	%
0	48	16	56	18.7	100	33.3	28	9.3	58	19.3
2	252	84	244	81.3	200	66.7	272	90.7	242	80.7

TABLE VI

Frequencies of Correct and Zero Scores-Question 6to10

Question	6		7		8		9		10	
Score	N	%	N	%	N	%	N	%	N	%
0	160	53.3	23	7.6	14	4.7	242	80.7	47	15.7
2	140	46.7	277	92.4	286	95.3	58	19.3	253	84.3

TABLE VII

Frequencies of Correct and Zero Scores-Questions 11to15

Question	11		12		13		14		15	
Score	N	%	N	%	N	%	N	%	N	%
0	16	5.4	56	18.7	30	10	17	5.7	80	26.7
2	284	94.6	244	81.3	270	90	283	94.3	220	73.3

TABLE VIII

Frequencies of Correct and Zero Scores-Questions 16to20

Question	16		17		18		19		20	
Score	N	%	N	%	N	%	N	%	N	%
0	169	56.3	171	57	17	5.6	155	51.7	59	19.7
2	131	43.7	129	43	283	94.4	145	48.3	241	80.3

TABLE IX

Frequencies of Correct and Zero Scores-Questions 21-25

Question	21		22		23		24		25	
Score	N	%	N	%	N	%	N	%	N	%
0	84	28	183	61	158	52.7	191	63.7	156	52
2	216	72	117	39	142	47.3	109	36.3	144	48

TABLE X

Frequencies of Correct and Zero Scores-Questions 26-30

Question	26		27		28		29		30	
Score	N	%	N	%	N	%	N	%	N	%
0	114	38	160	53.3	79	26.3	190	63.3	280	93.3
2	186	62	140	46.7	221	73.7	110	36.7	20	6.7

TABLE XI

Frequencies of Correct and Zero Scores-Questions 31-34

QUESTION	31		32		33		34		
Score	N	%	N	%	N	%	N	%	
0	85	28.3	161	53.7	102	34	62	20.7	
2	215	71.7	139	46.3	198	66	238	79.3	

Marking Scheme Question #35

(1) L.CD(finding L.CD enominators) - 1 mark.

(2) $r^2+8r+12<r^2+8r+16$ or alternate solution - 2 marks.

(3) $12<16$ - 1 mark.

TABLE XII

Frequencies of Correct and Zero Scores-Question 35

Scores	N	%
0	285	95.1
1	4	1.3
2	4	1.3
3	1	0.3
4	6	2.0

Marking Scheme Question #36

(1) $W=\frac{ktd^2}{l}$ - 1 mark

(2) Substitution - 1 mark

(3) k=1/10 - 1 mark

(4) $W=\frac{td^2}{10}$ - 1 mark

TABLE XIII

Frequencies of Correct and Zero Scores-Question 35

Scores	N	%
0	75	25.0
1	69	23.0
2	10	3.3
3	16	5.4
4	130	43.3

Marking Scheme Question #37

(a) Asymptotes at $x = 1$ and $x = -2$ ($\frac{1}{2}$ mark each) - 1 mark

(b) Behaviour of graph near Asymptotes-2marks($\frac{1}{2}$ mark each part)

(c) Intersection of graph with x axis -1 mark ($\frac{1}{2}$ mark each)

(d) Behaviour of graph for negative and positive values of x - 1 mark ($\frac{1}{2}$ mark each)

(e) If sketch is reasonably accurate - 1 mark

TABLE XIV

Frequencies of correct and Zero Scores-Question 37

Score	N	%
0	26	8.7
$\frac{1}{2}$	3	1.0
1	12	4.0
$1\frac{1}{2}$	14	4.7
2	16	5.3
$2\frac{1}{2}$	23	7.6
3	33	11.0
$3\frac{1}{2}$	20	6.6
4	15	5.0
$4\frac{1}{2}$	8	2.7
5	32	10.7
$5\frac{1}{2}$	6	2.0
6	92	30.7

Marking Scheme Question #38

½ mark for each part -

(a) $\rightarrow 2$ when $x \rightarrow \infty$

(b) $\rightarrow +\infty$ when $x \rightarrow 0$ provided $x > 0$

(c) Is never equal to $+2$

(d) $\rightarrow -\infty$ when $x \rightarrow 0$ provided $x < 0$

(e) Is equal to zero when $x = -\frac{1}{3}$

(f) $\rightarrow +2$ when $x \rightarrow -\infty$

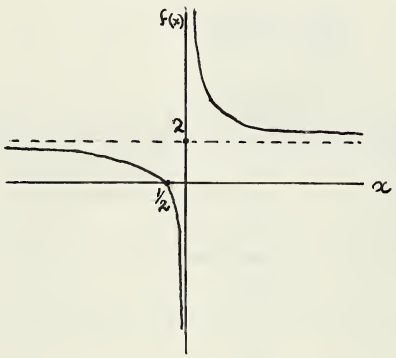


TABLE XV

Frequencies of Zero and Correct Scores-Question 38

Score	N	%
0	103	34.3
½	27	9.0
1	13	4.3
1½	16	5.4
2	22	7.3
2½	10	3.4
3	109	36.3

Marking Scheme Question #39

(1) $4x^2 + 5xy = 8$ $y = \frac{8-4x^2}{5x}$ - 2 marks(1 mark each)

(2) $V = \frac{x^2(8-4x^2)}{5x}$ or $\frac{8x-4x^3}{5}$ - 1 mark

(3) $\frac{dv}{dx} = \frac{8-12x^2}{5}$ - 1 mark

(4) Some proof to show $x = \sqrt{\frac{2}{3}}$ is the value which gives maximum - 1 mark.

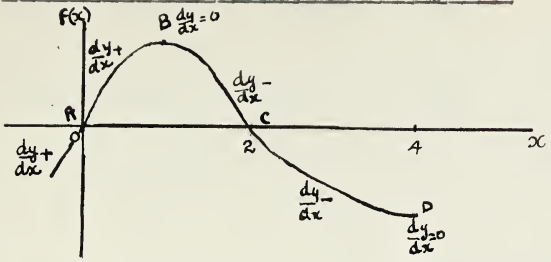
(5) Length of square edge $= \frac{2}{3}$ or 0.82 or 0.816 ft.-1 mark.

TABLE XVI

Frequencies of Zero and Correct Scores-Question 39

Scores	N	%
0	208	69.3
1	31	10.4
2	36	12.0
3	15	5.0
4	7	2.3
5	1	0.3
6	2	0.7

Marking Scheme - Question # 40



(1) One mark for each portion of the curve AB, BC, CD.

(2) One mark for positive slope at A (Slope not zero at origin).

(3) One mark for zero slope at B and D ($\frac{1}{2}$ mark each).

TABLE XVII

Frequencies of Scores - Question #40

Score	N	%
0	142	47.4
$\frac{1}{2}$	--	--
1	16	5.3
$1\frac{1}{2}$	5	1.6
2	9	3.0
$2\frac{1}{2}$	11	3.7
3	10	3.0
$3\frac{1}{2}$	12	4.0
4	7	2.3
$4\frac{1}{2}$	3	1.0
5	83	27.7

Marking Scheme - Question #41

(1) log 8132 + 2.5 log 4.65 - 2 log 325 - 2 marks.

(2) 3.9102 + 1.6688 - 5.0238 - 1 mark.

(3) anti log 0.5552 = 3.59 - 1 Mark.

TABLE XVIII

Frequencies of Scores - Question #41

Score	N	%
0	151	50.3
½	1	0.3
1	9	3.0
2	29	9.7
3	48	16.0
4	62	20.7

Marking Scheme - Question #42

(42a) For a change in x ,Δx,Δy is the corresponding change in y - 2 marks.

(42b) For two points on the curve, $\frac{\Delta y}{\Delta x}$ is the slope of the line joining these two points. - 1 mark.

(42c) For a point on the curve, $\frac{dy}{dx}$ is the slope of the tangent at that point. - 2 marks.

TABLE XIXa

Frequencies of Scores - Question 42a)

Scores	N	%
0	219	73.0
1	18	6.0
2	63	21.0

TABLE XIX b

Frequencies of Scores - Question 42b)

Scores	N	%
0	184	61.3
1	116	38.7

TABLE XIX c

Frequencies of Scores - Question 42c)

Scores	N	%
0	141	47.0
1	60	20.0
2	99	33.0

Marking Scheme - Question #43

(1) $\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{3}{2}}\right) + \frac{d}{dx}\left(x^{-\frac{1}{2}}\right)$ or $y = x^{\frac{3}{2}} + x^{-\frac{1}{2}}$ - 1 mark

(2) $= \frac{3x^{\frac{1}{2}}}{2} - \frac{1}{2}x^{-3/2}$ - 2 marks (one mark for each fraction)

(3) For $x = 4$, $\frac{dy}{dx} = 2\frac{15}{16}$.

TABLE XX

Frequencies of Scores - Question #43

Scores	N	%
0	168	56.0
1	54	18.0
2	5	1.7
3	18	6.0
4	9	3.0
5	46	15.3

Marking Scheme - Question #44

(1) $\pi \int_1^2 (x^2+1)^2 \, dx$ or $\pi \int_1^2 (x^4 + 2x^2 + 1) \, dx$ - 1mark

(2) $\pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x \right]_1^2$ - 2 marks

(3) $\frac{178}{15}\pi$ - 2 marks

(4) 37 cubic units - 1 mark

TABLE XXI

Frequencies of Scores - Question #44

Scores	N	%
0	152	50.7
1	25	8.3
2	29	9.7
3	28	9.3
4	29	9.7
5	17	5.7
6	20	6.6

Marking Scheme - Question #45

(1) $(x - 2) = (2x - 5)^2$ 1 mark

(2) $4x^2 - 21x + 27 = 0$ 1 mark

(3) $x = 3$ or $9/4$ 2 marks

(4) $9/4$ is an extraneous root 1 mark

TABLE XXII

Frequencies of Scores - Question #45

Score	N	%
0	228	76.0
1	17	5.7
2	18	6.0
3	4	1.3
4	28	9.3
5	5	1.7

Marking Scheme - Question # 46

(1) ar^{n-1} = 512 - 1 mark

(2) 2^{n-1} = 512 - 1 mark

(3) n = 10 - 1 mark

(4) One mark only for n = 10 by writing down the ten numbers.

TABLE XXIII

Frequencies of Scores - Question # 46

Scores	N	%
0	143	47.7
1	54	18.0
2	39	13.0
3	64	21.3

Marking Scheme - Question # 47

(1) a = 1 ; d = 1; n = 21 - 1 mark

(2) Summation - 1 mark

(3) Amount = $231.

(4) One mark only if all the terms are set down, and added.

TABLE XXIV

Frequencies of Scores - Question # 47

Scores	N	%
0	138	46.0
1	40	13.3
2	16	5.3
3	106	35.4

Marking Scheme - Question # 48

(1) $100 + 100(1.02) + 100(1.02)^2 + \ldots\ldots + (1.02)^{14}$.

or

$100 + 100(1.02) + 100(1.02)^2 + \ldots$ to 15 terms -2 Marks

(2) $100\left\{\frac{(1.02)^{15} - 1}{1.02 - 1}\right\}$ - 1 mark

(3) Amount = $1729.35 - 1 mark.

TABLE XXV

Frequencies of Scores - Question #48

Score	N	%
0	160	53.3
1	11	3.7
2	10	3.3
3	21	7.0
4	98	32.7

Marking Scheme - Question # 49

(1) $\frac{8!}{3!\ 3!\ 2!}$ 2 marks.

(2) 560 Signals 1 mark.

TABLE XXVI

Frequencies of Scores-Question #49

Scores	N	%
0	297	99.0
1	0	0.0
2	0	0.0
3	3	1.0

Marking Scheme - Question #50

(1) $(3^3)^{\frac{1}{3}}\left\{1 + \frac{1}{3^3}\right\}^{\frac{1}{3}}$ or $3\left(1+\frac{1}{27}\right)^{\frac{1}{3}}$ 2 marks

(2) $3\left(1 + \frac{1}{3}\cdot\frac{1}{3^3} + \cdots\cdots\right)$ or $3\left(1 + \frac{1}{81} + \cdots\cdots\right)$ 1 mark

(3) $\sqrt[3]{28} = 3.037$

TABLE XXVII

Frequencies of Scores - Question # 50

Score	N	%
0	215	71.7
1	13	4.3
2	13	4.3
3	9	3.0
4	50	16.7

Marking Scheme - Question # 51

(1) $\frac{dy}{dx} = 4 - 3x^2$ 1 mark

(2) $4 - 3x^2 = 0$ 1 mark

(3) $x = \pm \frac{2}{\sqrt{3}}$ or $\pm$ 1.55 1 mark

(4) Some proof to show that at $x = \frac{+2}{\sqrt{3}}$, function has a maximum value 1 mark

(5) Maximum value = 3.08 1 mark

TABLE XXVIII

Frequencies of Scores - Question # 51

Scores	N	%
0	62	20.7
1	29	9.7
2	104	34.7
3	67	22.3
4	26	8.6
5	12	4.0

Marking Scheme - Question # 52

(1) $\int_0^4 \sqrt{x}(4-x)\,dx$ - 2 marks.

(2) $\left[\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2}\right]_0^4$ 1 mark

(3) Substitution 1 mark

(4) Area = $8\frac{8}{15}$ or 8.53 sq. units 1 mark

TABLE XXIX

Frequencies of Scores - Question # 52

Scores	N	%
0	250	83.4
1	13	4.3
2	16	5.3
3	13	4.4
4	4	1.3
5	4	1.3

Marking Scheme - Question # 58

(1) Graphical Work - 2 marks

(2) Coefficient of $\sqrt{H}$ - 1 Mark

(3) Constant term is zero - but allow ± 1 - 1 mark

(4) $V = 8\sqrt{H}$ - 1 mark.

TABLE XXX

Frequencies of Scores - Question #53

Scores	N	%
0	71	23.3
1	46	15.3
2	81	27.0
3	38	12.7
4	30	10.0
5	35	11.7

II Graphs of Frequencies of Scores

The following graphs for questions 35 to 53 indicate clearly the distribution of scores on any one given test item.

Graphs for questions 1 to 34 are not included because the question in this objective part of the paper were either right or wrongi.e. there was no consideration given for a possibility that the candidate might have had an idea of the operation in the question considered.

N, as in Part I of this Chapter, refers to the number of candidates receiving a given score.

TABLE XXXI

Frequency table - Question # 35

Score	0	1	2	3	4
N	285	4	4	1	6

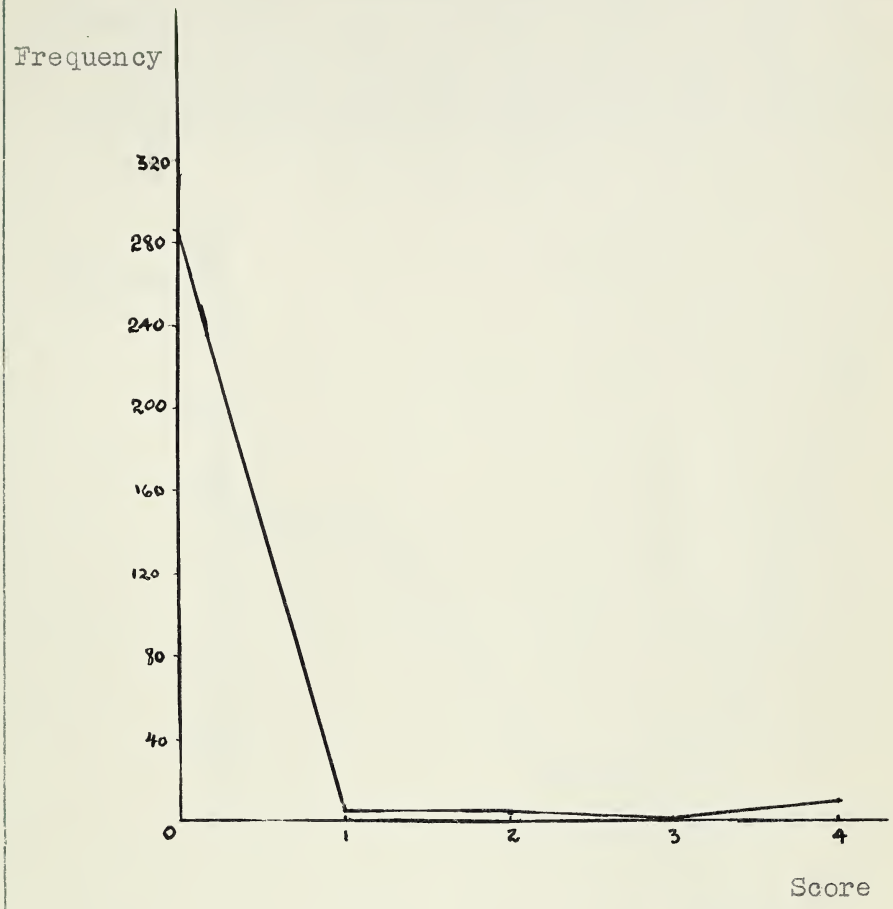


Fig. 2 Frequencies of Scores - Question # 35

TABLE XXXII

Frequency Table - Question # 36

Scores	0	1	2	3	4
N	75	69	10	16	130

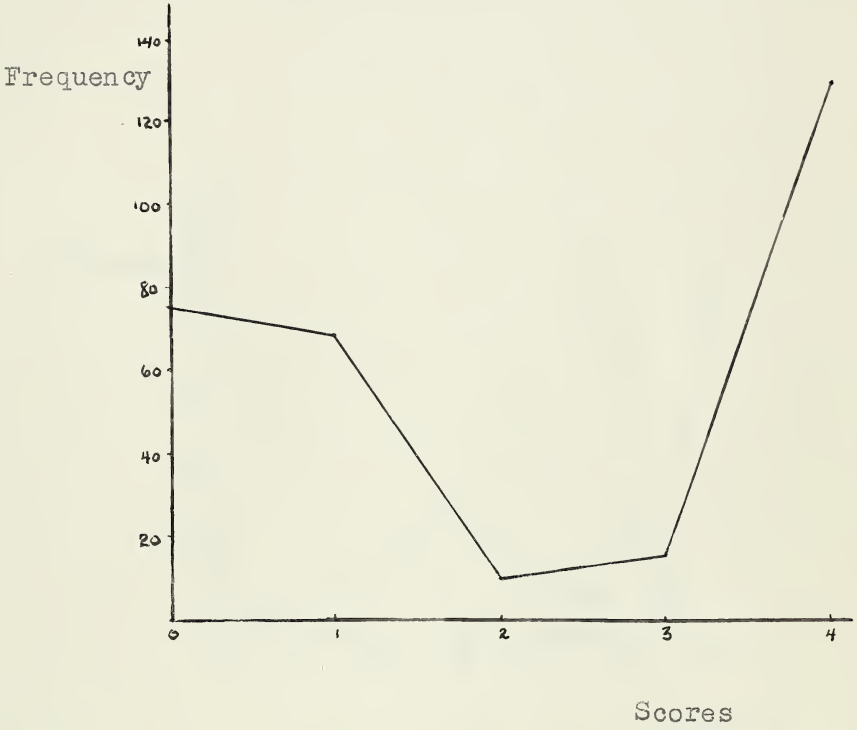


Fig. 3 Frequencies of Scores-Question # 36

TABLE XXXIII

Frequency Table - Question #37

Scores	0	½	1	1½	2	2½	3	3½	4	4½	5	5½	6
N	26	3	12	14	16	23	33	20	15	8	32	6	92

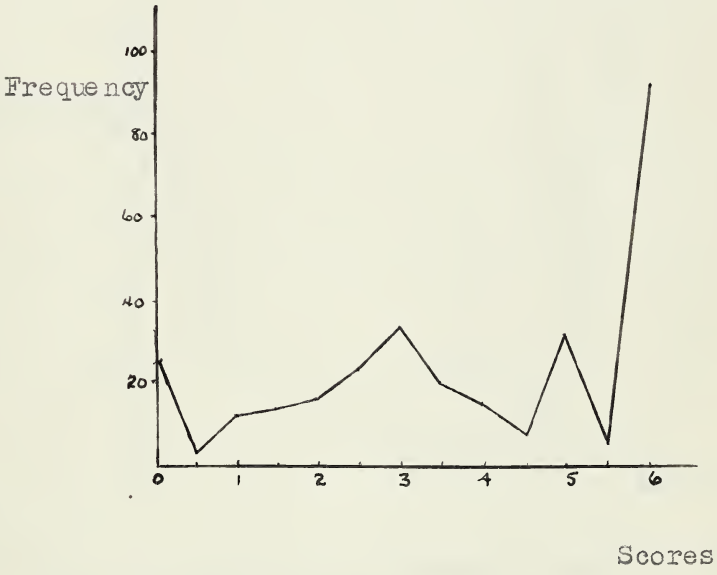


Fig. 4 Frequencies of Scores-Question 37

TABLE XXXIV

Frequency Table - Question # 38

Scores	0	½	1	1½	2	2½	3
N	103	27	13	16	22	10	109

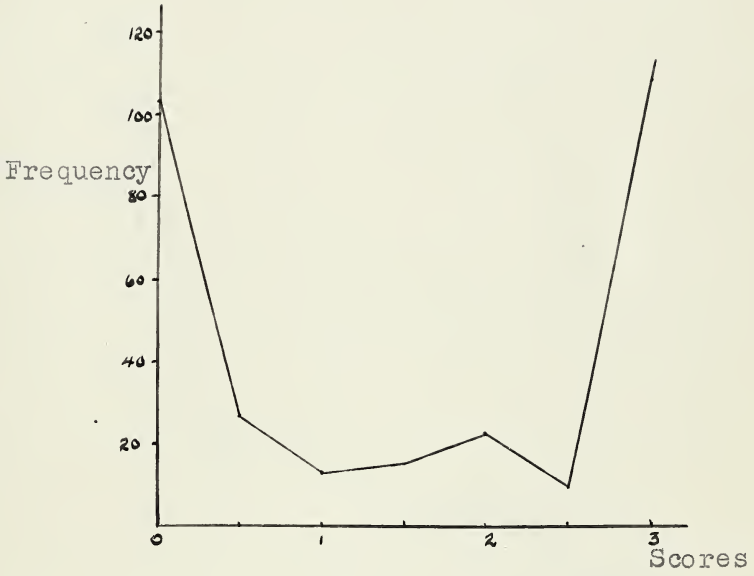


Fig. 5 Frequencies of Scores - Question #38

TABLE XXXV

Frequency Table ---- Question # 39

Scores	0	1	2	3	4	5	6
N	208	31	36	15	7	1	2

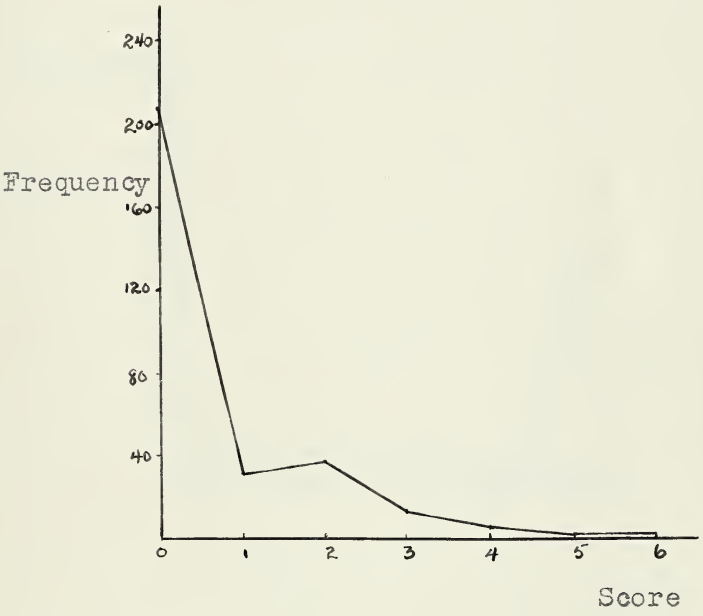


Fig. 6 Frequencies of Scores-Question # 39

TABLE XXXVI

Frequency Table - Question # 40

Scores	0	½	1	1½	2	2½	3	3½	4	4½	5
N	142	0	16	5	9	11	10	12	7	3	83

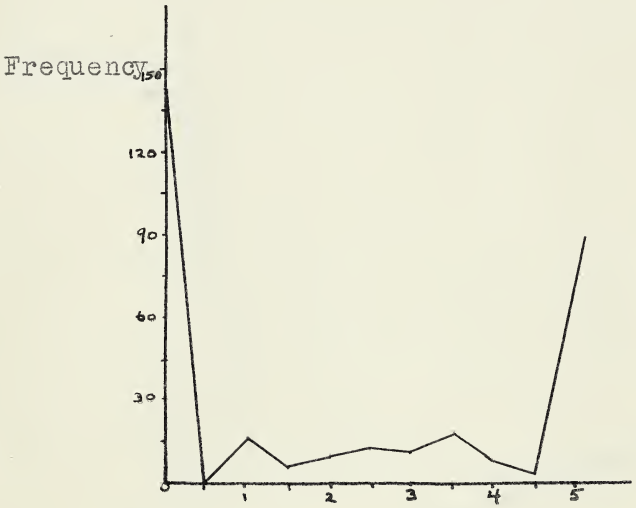


Fig. 7 Frequencies of Scores-Question #40

TABLE XXXVII

Frequency Table - Question # 41

Scores	0	½	1	2	3	4
N	151	1	9	29	48	62

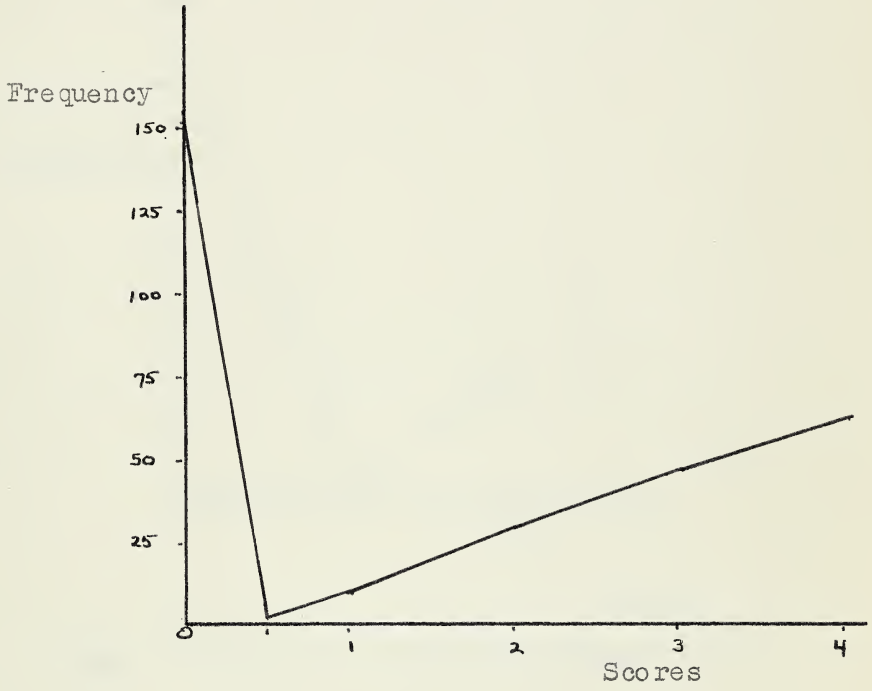


Fig. 8 Frequencies of Scores-Question 41

TABLE XXXVIIIa

Frequency Table - Question 42a

Scores	0	1	2
N	219	18	63

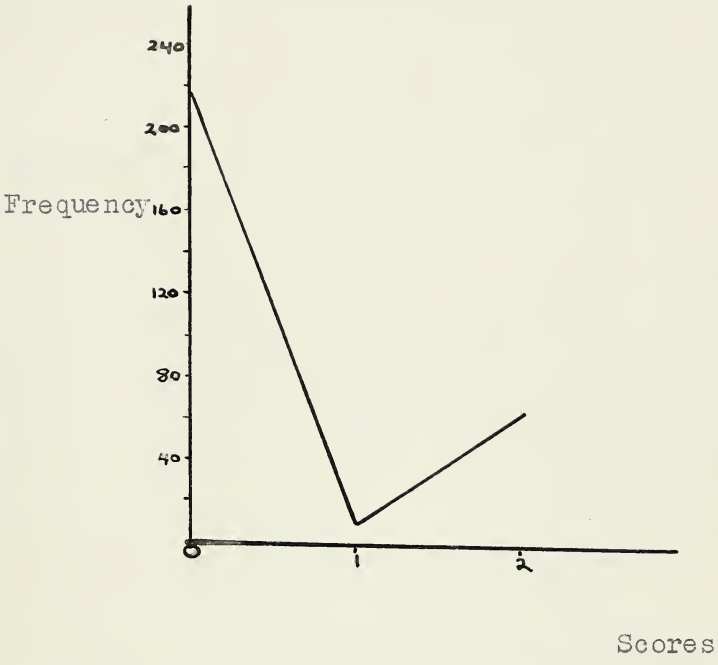


Fig. 9 Frequencies of Scores - Question 42a

TABLE XXXVIII

Frequency Table - Question # 42c

Scores	0	1	2
N	141	60	99

Fig. 10 Frequencies of Scores - Question 42c

TABLE XXXIX

Frequency Table - Question # 43

Score	0	1	2	3	4	5
N	168	54	5	18	9	46

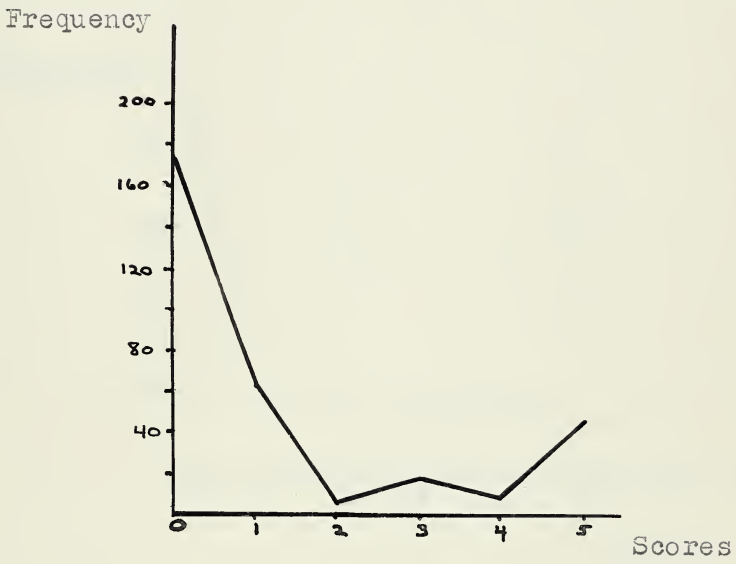


Fig. 11 Frequencies of Scores-Question 43

TABLE XL

Frequency Table - Question #44

Scores	0	1	2	3	4	5	6
N	152	25	29	28	29	17	20

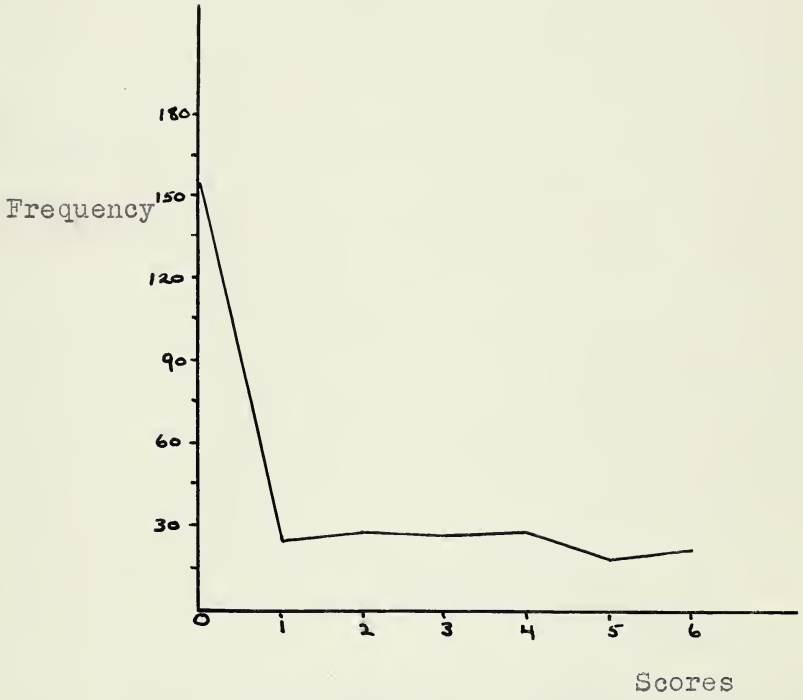


Fig. 12 Frequencies of Scores -Question 44

TABLE XLI

Frequency Table - Question # 45

Score	0	1	2	3	4	5
N	228	17	18	4	28	5

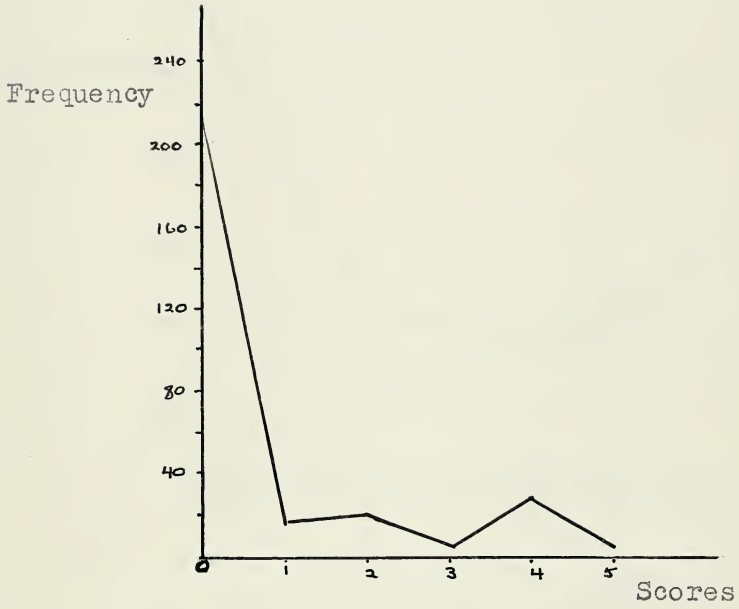


Fig. 13 Frequencies of Scores-Question 45

TABLE XLII

Frequency Table - Question # 46

Scores	0	1	2	3
N	143	54	39	64

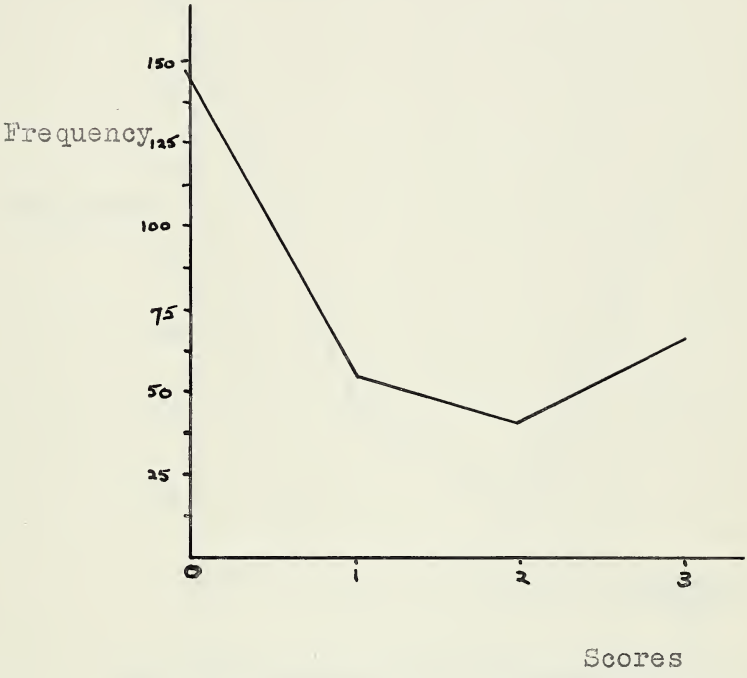


Fig. 14 Frequencies of Scores-Question 46

TABLE XLIII

Frequency Table - Question #47

Scores	0	1	2	3
N	138	40	16	106

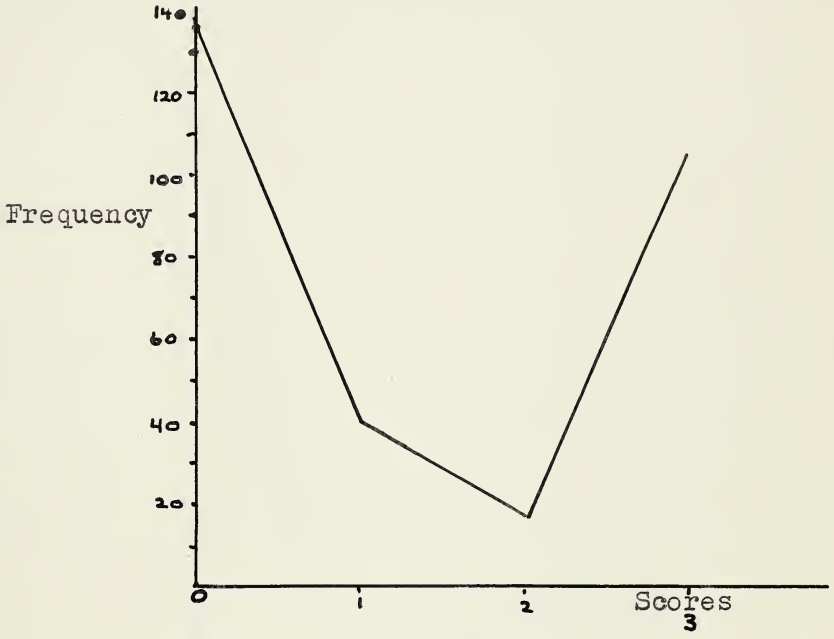


Fig. 15 Frequencies of Scores-Question 47

TABLE XLIV

Frequency Table - Question # 48

Scores	0	1	2	3	4
N	160	11	10	21	98

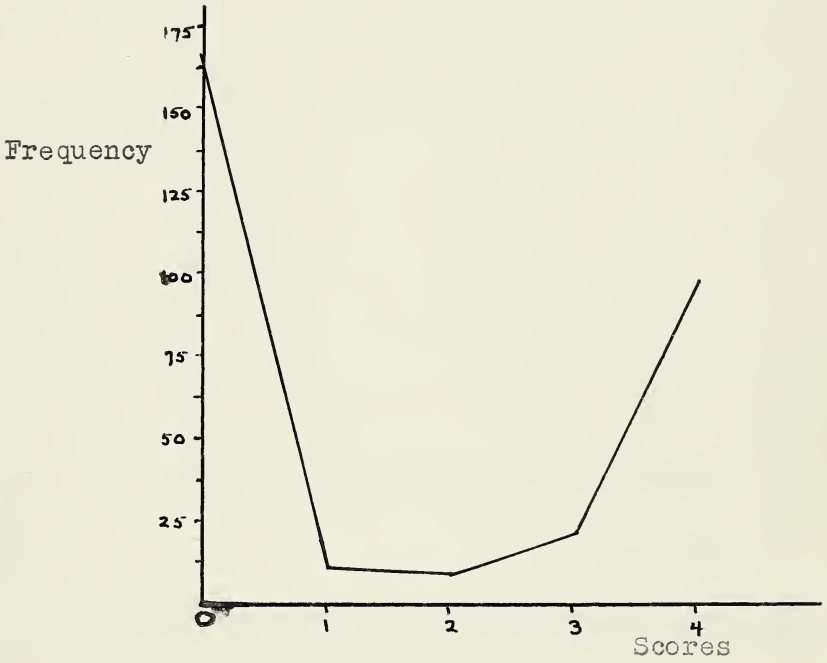


Fig. 16 Frequencies of Scores-Question 48

TABLE XLV

Frequency Table - Question #49

Scores	0	1	2	3
N	297	0	0	3

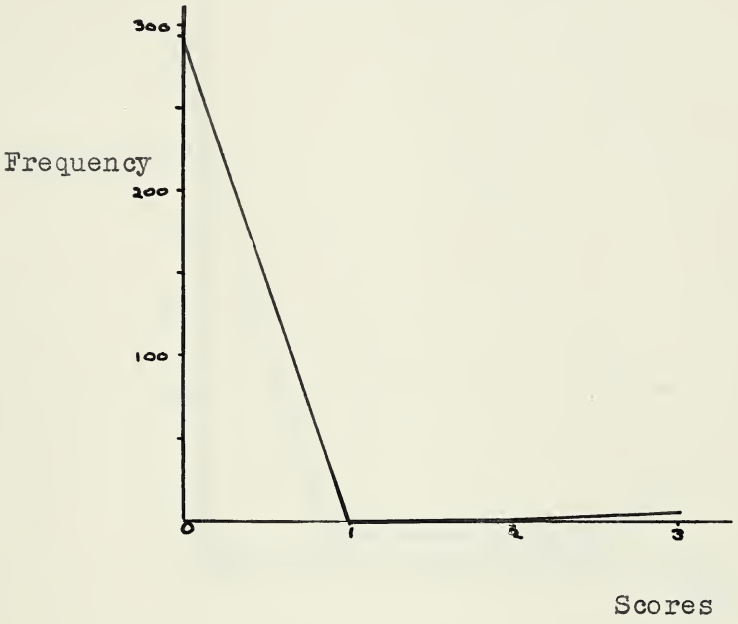


Fig. 17 Frequencies of Scores-Question 49

TABLE XLVI

Frequency Table - Question #50

Scores	0	1	2	3	4
N	215	13	13	9	50

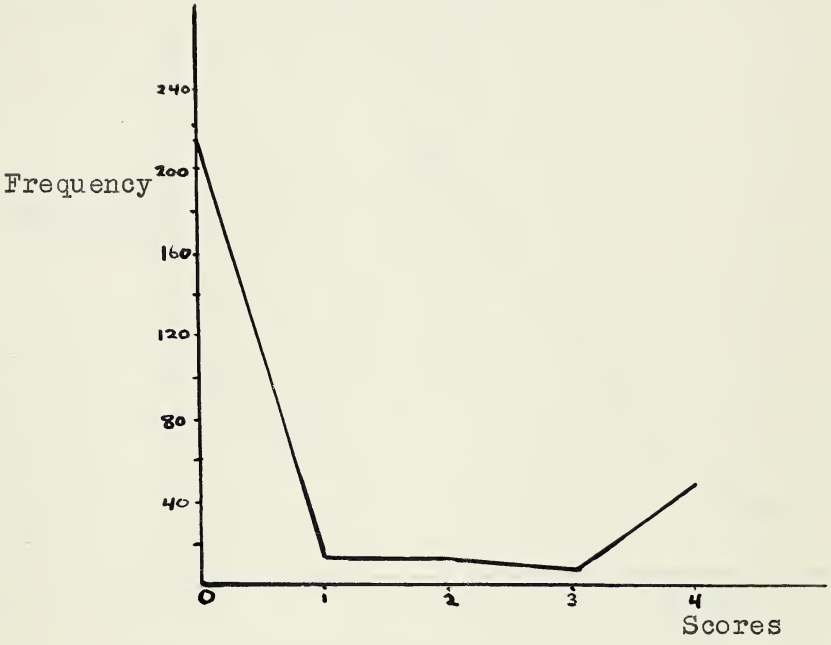


Fig. 18 Frequencies of Scores-Question 50

TABLE XLVII

Frequency table - Question #51

Scores	0	1	2	3	4	5
N	62	29	104	67	26	12

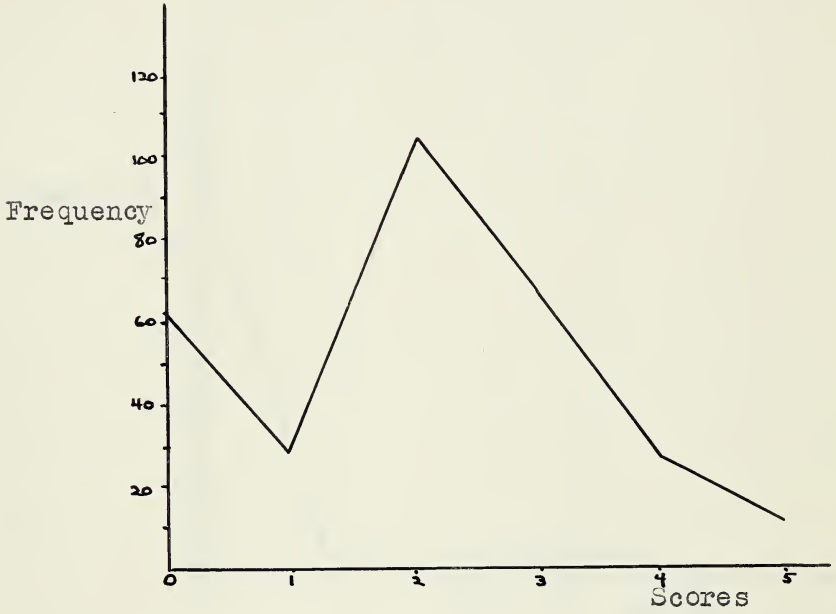


Fig. 19 Frequencies of Scores-Question 51

TABLE XLVIII

Frequency Table - Question # 52

Scores	0	1	2	3	4	5
N	250	13	16	13	4	4

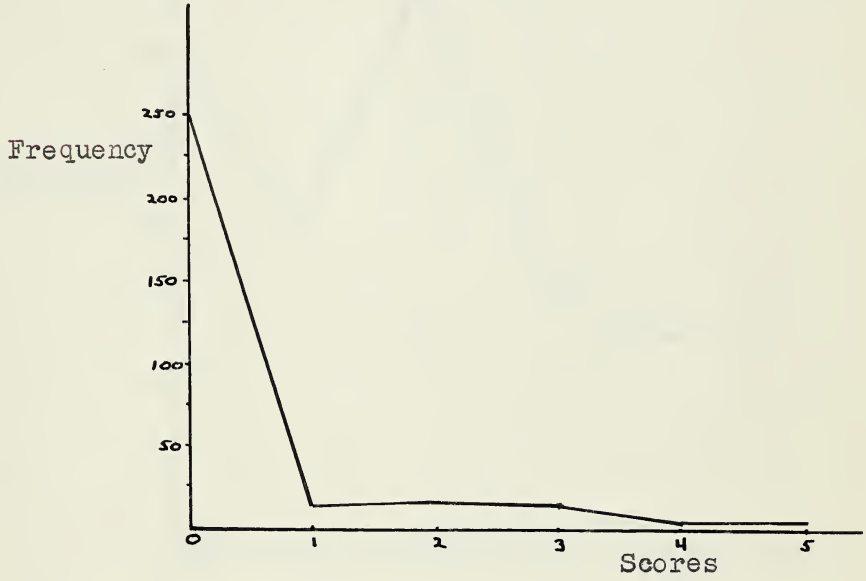


Fig. 20 Frequencies of Scores-Question 52

TABLE XLIX

Frequency Table - Question # 53

Scores	0	1	2	3	4	5
N	70	46	81	38	30	35

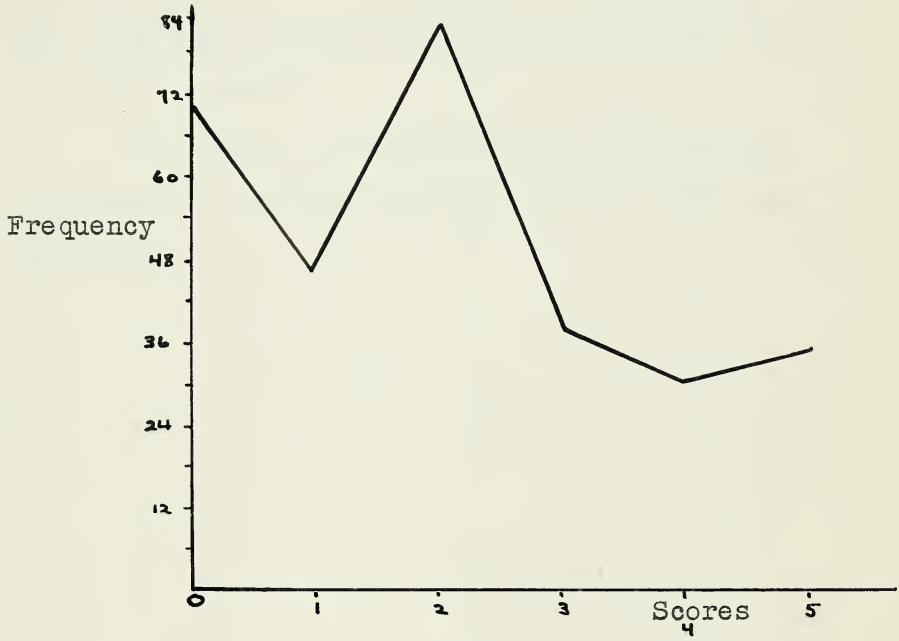


Fig. 21 Frequencies of Scores-Question 53

III Frequencies and Graphs of Perfect Scores

The following graphs and frequency tables illustrate quite forcibly the ability of this group. Their ability at problem solving is very low. However, the ability to select an answer from a given set of answers poses a relatively simple problem for these students.

As before, N represents the total number of candidates making a perfect score on any one question.

TABLE L

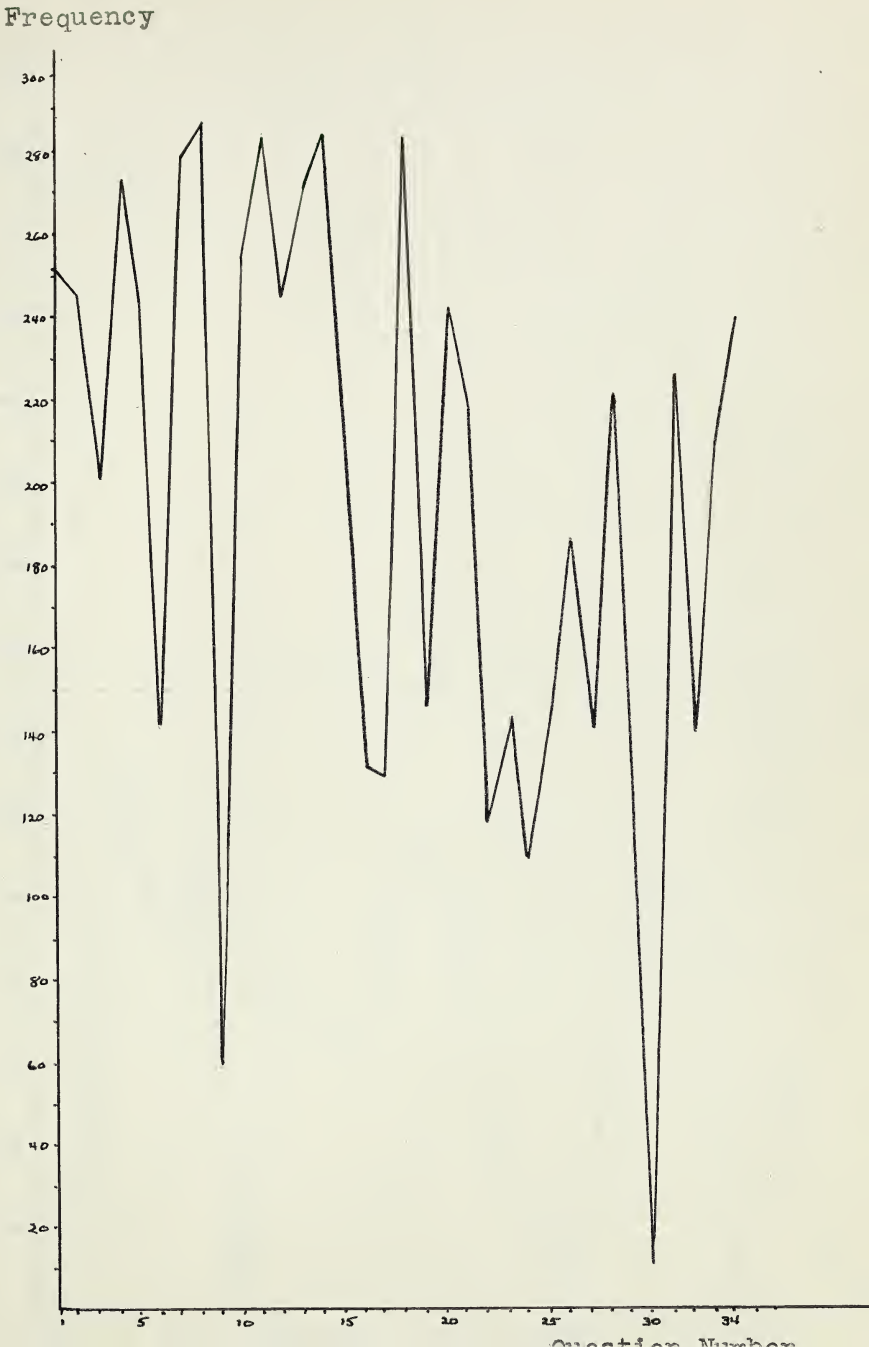
Frequencies of Perfect Scores-Question 1-26

Question	N	%
1	252	84.0
2	244	81.3
3	200	66.7
4	272	90.7
5	242	80.7
6	140	46.7
7	277	92.4
8	286	95.3
9	58	19.3
10	253	84.3
11	284	94.6
12	244	81.3
13	270	90.0
14	283	94.3
15	220	73.3
16	131	43.7
17	129	43.0
18	283	94.3
19	145	48.3
20	241	80.3
21	216	72.0
22	117	39.0
23	142	47.3
24	109	36.3
25	144	48.0
26	186	62.0
27	140	46.7
28	221	73.7
29	110	36.7
30	20	6.7
31	215	71.7
32	139	46.3
33	198	66.0
34	238	79.3

TABLE LI

Frequencies of Perfect Scores- Questions 35-53

Question	N	%
35	6	2.0
36	130	43.3
37	92	30.7
38	109	36.3
39	2	0.7
40	83	27.7
41	62	20.7
42a	63	21.0
42b	116	38.7
42c	99	33.0
43	46	15.3
44	20	6.6
45	5	1.7
46	64	21.3
47	106	35.4
48	98	32,7
49	3	1.0
50	50	16.7
51	12	4.0
52	4	1.3
53	35	11.7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280
260
240
220
200
180
160
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100
80
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25
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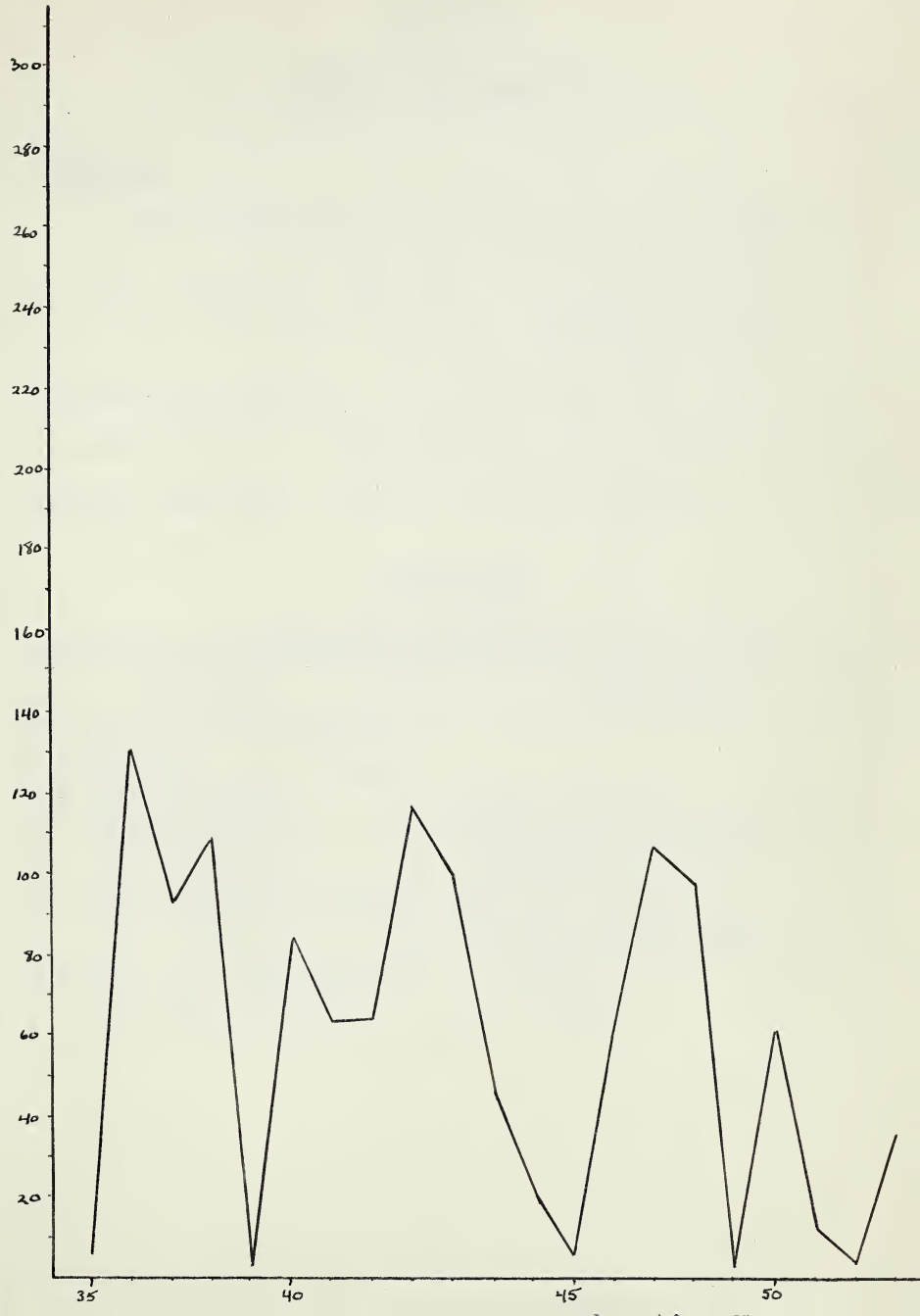
34
Question Number


Frequency
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280
260
240
220
200
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120
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35
40
45
50

CHAPTER V

Negative Analysis

I Errors

The mechanical errors made by the candidates of this group were many and varied. In the following table the errors have been classified under appropriate headings. The following examples, taken directly from the answer papers, will serve to illustrate the kind of errors which were made.

TABLE LII

Examples of Errors

Category	Examples
I Arithmetic of small numbers	
a. Addition and Subtraction	$93+70+15=188$; $2-\frac{1}{2}=1$
b. Multiplication and Division	$27\times16=412$; $5\times16=90$; $24/8=4$.
c. Extraction of square and cube roots	$\sqrt{4/9}=\frac{2}{3}$; $\sqrt{1.33}=0.12$; $\sqrt{3}=1.5$; $27^{1/3}=9$; $\sqrt{-48}=-6.92$
d. Did not place $\pm$ sign in front of root	$x^2=3$; $x=\sqrt{3}$

cont'd.

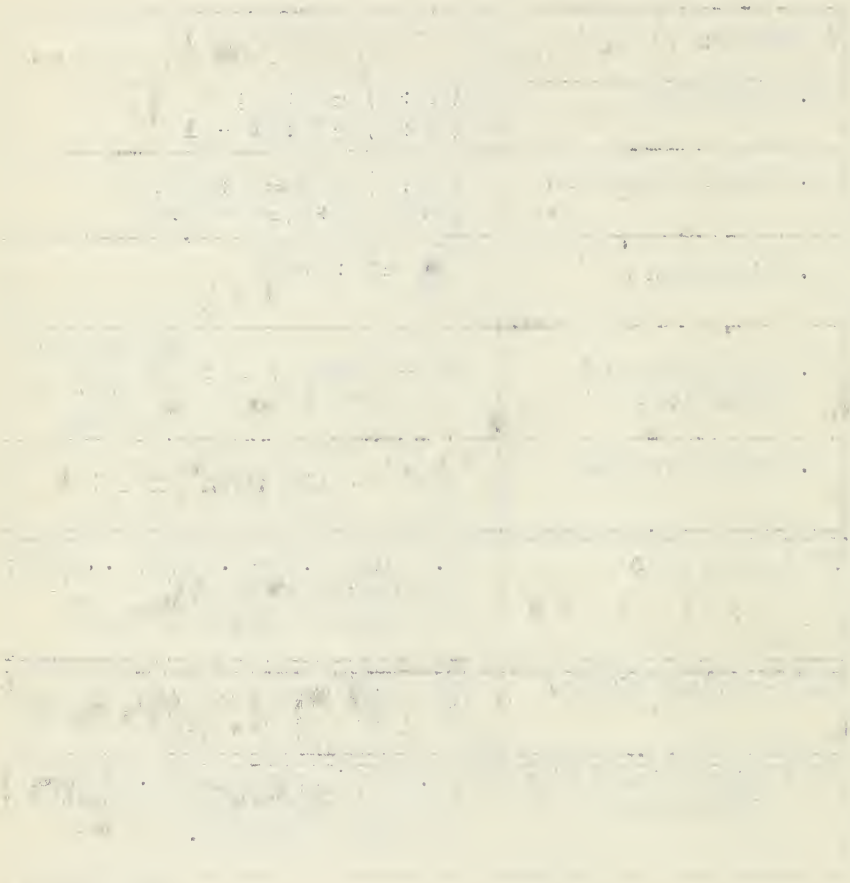
TABLE LII - cont'd.

II Fractions	
a. Multiplication and Division	1/3x1/27= 1/74; 21/2= $11\frac{1}{2}$
b. Addition and Subtaction	96/15+20/15=181/15; 13 11/15-1 13/15= 12 7/15
c. Relation between decimals and fractions.	$2.5 = \frac{3}{2}$
III Decimals	
a. Addition and Subtraction	3.9102+1.6688=4.5790; 5.5790-5.0238=0.4452
b. Multiplication and Division	2.5119x2=3.0238; $\frac{3.1416}{2} = 1.0472$
IV Indices	
a. Multiplication	$(\sqrt{3})^3 = 9$; $(3)^3 = 9$
b. Division	$(x-2)^{-1} = x+2$; $\frac{1}{\sqrt{x}} = x^{\frac{1}{2}}$; $\frac{1}{\sqrt{x}} = x^{-1}$; $\frac{3}{2\sqrt{x}} = \sqrt{1.5x}$

TABLE LII - cont'd

V Albegra	
a. Squaring	$(x^2+1)^2=x^4+1$; $(x^2+1)^2=x^4+2x^3-1$
b. Multiplication	$(x^2+1)\ x^{-1/2}=x^2+x^{-1/2}$; $(r+2)\ (r+6)=r^2-8r+12$
c. Division	$\frac{x^2+1}{\sqrt{x}}=x^2+1+x^{-1/2}$
d. Solutions of Equations	$2x^3=2^3$ when $x=1$; $8/5-\frac{12}{5}x^2=0$ $8-12x^2=5$ $x=\sqrt{3}/2$
e. Substitution	$\frac{1}{5}x^5=12\frac{4}{5}$ when $x=2$
VI Logarithms	2.5 log 4.65=1.6678(1.6688); antilog 0.552 = 3591.00; 2.5 log 4.65 = log 2.5 x log 4.65
VII Copying Errors	$\frac{3\sqrt{x}}{2}=\frac{2\sqrt{x}}{2}$; $\frac{1}{3}=\frac{1}{5}$; $\frac{1}{3}=\frac{2}{3}$; $\frac{80}{3}=\frac{80}{5}$
VIII Significant Figures in answers	3.591 instead of 3.59 when answer was asked "correct to two decimals."

concluded.



The following table shows the frequency of errors which candidates made on the paper. It may be noted here that the frequency of mechanical errors is based on the written work which the candidate did on questions 35 - 53, inclusive. Of these 19 questions, questions number 37, 38 and 40 are graph sketching problems, while question 42 is a question requiring written statements. Thus for these four questions no mathematical manipulation was necessary. In reality, then, the frequency of errors is based on 15 questions.

TABLE LIII

Frequencies of Mechanical Errors

Category	N	%
I. Arithmetic of small numbers		
a. Addition & Subtraction	11	1.3
b. Multiplication & Division	116	13.9
c. Extraction of Square & Cube Roots	25	3.0
d. Failure to place ± sign before root	46	5.5

cont'd.

TABLE LIII
concluded.

Category	N	%
II Fractions		
a. Addition & Subtraction	3	0.4
b. Multiplication & Division	12	1.6
III. Decimals		
a. Addition & Subtraction	14	1.7
b. Multiplication & Division	22	2.6
c. Relation between decimals and fractions	1	0.1
IV. Indices		
a. Multiplication & Division	127	15.2
V. Algebra		
a. Squaring	11	1.3
b. Multiplication & Division	143	17.1
c. Solutions of Equations	71	8.5
d. Substitutions	18	2.4
VI. Logarithms	74	8.9
VII. Copying Errors	96	11.4
VIII. Miscellaneous		
a. Significant Figures	43	5.1
TOTALS	833	100.0

The results show clearly that mechanical errors are indeed the stumbling block to success, at least for this group. Further comments and recommendations will be given in the chapter dealing with conclusions.

II ERRORS

A) Questions 1 - 34

The following tables indicate the errors which were made on the different test items. As has been previously stated questions 1 - 34 are of the multiple choice type and choices other than the correct one have beencounted as errors.

In each case, the question will be given and following this, the correct solution as set down by the Examinations Board.

Question 1. The value of X° is

a. o b. undefined

c. $\frac{1}{x}$ d. x

e. 1.

Correct Solution : e. 1.

TABLE LIV

Frequencies of Errors, Correct answers & No attempts.
Question 1.

	N	%
Correct Answer	252	84.0
No Attempt	2	0.7
Gave answer as a. o	21	7.0
Gave answer as b. undefined	21	7.0
Gave answer as c. $\frac{1}{x}$	0	0.0
Gave answer as d. x	4	1.3

On the whole Question 1 was very well done. However, there is no reason for answers which 46 candidates gave. A rigorous proof for this theorem in indices is given on page 163 of the Grade X "Algebra for to-day". A speculative reason for the response b. undefined can be given. In work on limits, the term undefined has a definite meaning and after learning this, some students become conditioned to saying "undefined" whenever they encounter the symbol o.

Question 2. The fourth proportional to 2, 3 and 4 is a. 5; b. 6; c. 7; d. 9; e. 24.

Correct Solution : b. 6.

TABLE LV

Frequencies of Errors, Correct answers & No attempts. Question 2.

	N	%
Correct Answer	244	81.3
No Attempt	0	0.0
Gave answer as a. 5	55	18.3
Gave answer as c. 7	0	0.0
Gave answer as d. 9	1	0.4
Gave answer as e. 24	0	0.0

The results show that this question has been very well done. The same question appears on page 6 of the text Senior Algebra. The 55 candidates giving a. 5 as a solution were no doubt confused as to the difference between fourth proportional and continued proportion. This problem is one of two which was attempted by all candidates. The other question, on the paper, which everyone attempted was question 8.

Question 3. Which ratio has the greatest value: a. b. c. d. or e. ?

a. $\frac{11}{12}$, b. $\frac{47}{48}$, c. $\frac{5}{6}$, d. $\frac{23}{24}$, c. $\frac{35}{36}$

Correct Solution: b. $\frac{47}{48}$

TABLE LVI

Frequencies of Errors, Correct answers & No attempts.
Question 3

	N	%
Correct Answer	200	66.7
No Attempts	1	0.3
Gave answer as a. 11/12	3	1.0
Gave answer as c. 5/6	81	27.0
Gave answer as d. 23/24	2	0.7
Gave answer as e. 35/36	13	4.3

The pupil first meets the term ratio in Grade VII. Page 28 of the Grade VII "Mathematics for Everyday Use" deals with comparison of numbers by division. This question was only fairly well done and the results indicate a weak background in Grade VII fundamentals.

Question 4. If the radii of two circles are to each other as 2:3 then the areas of the two circles are to each other as : a. 4:9; b. 2:3; c. 3:5; d. 5:2; e. 5:3. Correct Solution a. 4:9

TABLE LVII

Frequencies of Errors, Correct answers & No attempts.

Question 4		
	N	%
Correct Answer	272	90.7
No Attempt	3	1.0
Gave answer as b. 2:3	23	7.7
Gave answer as c. 3:5	1	0.3
Gave answer as d. 5:2	1	0.3
Gave answer as e. 5:3	0	0.0

This question was very well done. Question 16, Page 3 of the text is very similar.

Question 5. The graph of the function $(x+2)(x-3)$ crosses the y axis at a. 3, b. 2, c. 6, d. -1, e. -6.

Correct Solution : e. -6

TABLE LVIII

Frequencies of Errors, Correct answers & No attempts.
Question 5

	N	%
Correct Answer	242	80.7
No Attempt	1	0.3
Gave answer as a. 3	43	14.4
Gave answer as b. 2	0	0.0
Gave answer as c. 6	7	2.3
Gave answer as d. -1	7	2.3

This question was very well done. The results indicate that pupils of this group have a grasp of one of the fundamentals prerequisite to sketching the graph of a function.

Question 6. The area of a circle as a function of its diameter (D) is: a. πD, b. $\frac{\pi D}{2}$, c. πD^2, d. $\frac{\pi D^2}{4}$, e. $\frac{\pi D^2}{2}$

Correct Solution d. $\frac{\pi D^2}{4}$.

TABLE LIX

Frequencies of Errors, Correct answers & No attempts.
Question 6

	N	%
Correct Answer	140	46.7
No Attempt	6	2.0
Gave answer as a.	56	18.7
Gave answer as b.	11	3.6
Gave answer as c.	9	3.0
Gave answer as e.	78	26.0

This question was poorly done. On the whole pupils in the score range under consideration show limited ability in expressing functional relationships.

Question 7. The value of w in the formula $P=\frac{Q+nw}{8}$ is:

a. 8P-nw, b. $\frac{8P-Q}{n}$, c. $\frac{P-Q}{8n}$, d. $\frac{8P+nw}{n}$, e. $\frac{8(P-Q)}{n}$

Correct Solution: b. $\frac{8P-Q}{n}$

TABLE LX

Frequencies of Errors, Correct answers & No attempts.

Question 7		
	N	%
Correct Answer	277	92.4
No Attempt	2	0.7
Gave answer as a. 8P-nw	0	0.0
Gave answer as c. $\frac{P-Q}{8n}$	7	2.3
Gave answer as d. $\frac{8P+nw}{n}$	1	0.3
Gave answer as e. $\frac{8(P-Q)}{n}$	13	4.3

This question was very well done. A great deal of similar work is done in Grade IX. A Problem exactly like this one, but with different wording as to directions, is question 2 Ex. E. page 155 of the text "Senior Algebra".

Question 8. If $f(x) = x^3 - x^2$, then the value of $f(-2)$ is equal to: a. o, b. 12, c. 4, d. -12, e. -8.

Correct Solution: d. -12

TABLE LXI

Frequencies of Correct Answers, No Attempts & Errors
Question 8

	N	%
Correct Answer	286	95.3
No Attempt	0	0.0
Gave answer as a. o	0	0.0
Gave answer as b. 12	1	0.3
Gave answer as c. 4	11	3.7
Gave answer as e. -8	2	0.7

As results indicate pupils experienced little difficulty in handling functional notation. It is significant to note here that this problem was attempted by everyone. Problems similar to this one appear on P. 41-42 of the prescribed text.

Question 9. If h=o then the value of $\frac{2h + h^2}{h}$ is

a. 2+h, b. undefined, c. 2, d. o , e. ∞

Correct Solution : b. undefined

TABLE LXII

Frequencies of Correct Answers, No Attempts & Errors
Question 9

	N	%
Correct Answer	58	19.3
No Attempt	2	0.7
Gave answer as a. 2+h	8	2.7
Gave answer as c. 2	120	40.0
Gave answer as d. 0	96	32.0
Gave answer as e. ∞	16	5.3

The indicated results show a very weak grounding in the theory of Limits. Those who gave as an answer - c. 2 misread the problem. This question appears in the discussion of Example II on page 47 of the prescribed text. The only difference being that instead of "undefined," the term "meaningless" is used.

Question 10. If the graph of the equation $3x+py=12$ passes through the point $(-2,4.5)$, the value of p is a. 2, b. 6, c. 4, d. 2.5, e. -2

Correct Solution : c. 4

TABLE LXIII

Frequencies of Correct Answers, No attempts & Errors Question 10

	N	%
Correct Answer	253	84.3
No Attempt	17	5.7
Gave answer as a. 2	3	1.0
Gave answer as b. 6	7	2.3
Gave answer as d. 2.5	14	4.7
Gave answer as e. -2	6	2.0

Candidates experienced little difficulty with this question. Problems similar to this one appear on page 67 of the Grade X text "Algebra for To-Day".

Question 11. M varies directly as s. If M=40 when s=12, then when s=18, M equals a 6, b. 30, c. 60, d. 5.4, e. 26 2/3.

Correct Solution: c. 60

TABLE LXIV

Frequencies of Correct Answers, No Attempts & Errors. Question 11

	N	%
Correct Answer	284	94.6
No Attempt	1	0.3
Gave answer as a. 6	2	0.7
Gave answer as b. 30	2	0.7
Gave answer as d. 5.4	9	3.0
Gave answer as e. 26 2/3	2	0.7

This question was exceptionally well done.

Question 12. The gradient of the straight line which passes through the points (-4,6) and (3,-2) is a. -7/8, b. $\sqrt{17}$, c. $\sqrt{65}$, d. -8/7, e. 8/7

Correct Solution: d. -8/7

TABLE LXV

Frequencies of Correct Answers, No Attempts & Errors Question 12

	N	%
Correct Answer	244	81.3
No Attempt	9	3.0
Gave answer as a. -7/8	30	10.0
Gave answer as b. $\sqrt{17}$	0	0.0
Gave answer as c. $\sqrt{65}$	0	0.0
Gave answer as e. 8/7	17	5.7

This question was well done. The student first meets problems of this type in Grade X Algebra. Surprising enough is the fact that 3% of the candidates had no idea of how to find a gradient. 10% of the candidates took the gradient to be $\frac{x-x_1}{y-y_1}$ instead of $\frac{y-y_1}{x-x_1}$ while 5.7% omitted the sign.

Question 13. A simple function which has the following properties $f(2) = 0$ and $f(-3) = 0$ is a. x^2+5x+6, b. x^2-x+6, c. x^2+x-6, d. x^2-3x+2, e. x^2+x+6

Correct Solution: c. x^2+x-6

TABLE LXVI

Frequencies of Correct Answers, No Attempts & Errors Question 13

	N	%
Correct Answer	270	90.0
No Attempt	6	2.0
Gave answer as a. x^2+5x+6	2	0.7
Gave answer as b. x^2-x+6	11	3.7
Gave answer as d. x^2-3x+2	6	2.0
Gave answer as e. x^2+x+6	5	1.6

This question was very well done by candidates of this group. Pupils showed that they had a grasp of functional notation and the ability to apply this language.

Question 14. $\frac{d}{dx}(2x-4)(x+3)$ is a. 2, b. $4x+2$, c. $2x^2$, d. $(2x+2)$, e. $4x$

Correct Solution b. $4x+2$

TABLE LXVII

Frequencies of Correct Answers, No Attempts & Errors.
Question 14

	N	%
Correct Answer	283	94.3
No Attempt	1	0.3
Gave answer as a. 2	10	3.4
Gave answer as c. $2x^2$	0	0.0
Gave answer as d. $2x+2$	5	1.7
Gave answer as e. $4x$	1	0.3

This problem was exceptionally well done. 3.4 % of the candidates did not multiply $(2x-4)(x+3)$ and instead differentiated each term individually, then multiplied. A problem similar to this one is # 27 on page 69 of the text.

Question 15. The cube of $3x^2$ divided by the square of $2x^2$ is a. $\frac{27}{4}x^2$, b. $\frac{27x}{4}$, c. $\frac{27x^4}{4}$, d. $\frac{9x}{4}$, e. $\frac{9x^4}{4}$

Correct Solution a. $\frac{27x^2}{4}$

TABLE LXVIII

Frequencies of Correct Answers, No Attempts & Errors Question 15

	N	%
Correct Answer	220	73.3
No Attempt	3	1.0
Gave answer as b. $\frac{27x}{4}$	31	10.3
Gave answer as c. $\frac{27x^4}{4}$	32	10.7
Gave answer as d. $\frac{9x}{4}$	8	2.7
Gave answer as e. $\frac{9x^4}{4}$	6	2.0

As results indicate this question was only fairly well done. Problems of this type are found in the Grade X Algebra Text. In reality then this problem is a review problem, for grade XII pupils, in the language of indices. This problem was taken directly from the Revision Exercises in the text, where it is question #1 in Exercise A. The poor showing indicates a general lack of knowledge concerning indices. Table LII further bears out this fact.

Question 16. Log $\frac{r}{s}$ + log s may be written:

a. Log $\frac{r+s}{s}$, b. log rs, c. log $\frac{r-s}{s}$, d. log r, e. log $\frac{r}{s^2}$

Correct Solution:d.log r

TABLE LXIX

Frequencies of Correct Answers, No Attempts & Errors
Question 16

	N	%
Correct Answers	131	43.8
No Attempts	24	8.0
Gave answer as a. log $\frac{r+s}{s}$	105	35.0
Gave answer as b. log rs	12	4.0
Gave answer as c. log $\frac{r-s}{s}$	14	4.6
Gave answer as e. log $\frac{r}{s^2}$	14	4.6

The results clearly indicate the weakness of candidates in regard to manipulating logarithmic equations. The topic of logarithms is optional in Grade X. As a result some students come to Grade XII Math without even the basic rudimentary notions of Logarithms. This problem was taken directly from the text, where it appears as problem 10, page 155, Revision Exercise D.

Question 17. 0.0000716 may be written a. 7.16×10^{-6}, b. 7.16×10^{-5}, c. 716×10^{-9}, d. 7.16×10^{-7}, e. 716×10^{-5}.

Correct Solution: b. 7.16×10^{-5}

TABLE LXX

Frequencies of Correct Answers, No Attempts & Errors. Question 17

	N	%
Correct Answer	129	43.0
No Attempt	14	4.7
Gave answer as a. 7.16×10^{-6}	35	11.7
Gave answer as c. 7.16×10^{-9}	3	1.0
Gave answer as d. 7.16×10^{-7}	30	10.0
Gave answer as e. 7.16×10^{-5}	89	29.6

Results for this question indicate a weak background in the use of index notation. Students are introduced to the index notation on p. 169 of the Grade IX text. This work in indices is further amplified on p. 167 of the Grade X text. Problem 6 on Page 167 of t Grade X text has a problem similar to this one. On p. 157 of the Grade XII text, problem 2 in Revision Ex. F is very much similar. Thus there should be no reason whatever for the weak showing of the candidates on this type of problem.

Question 18. If $y = x(4x^2-7)$, then $\frac{dy}{dx}$ is equal to

a. $12x^2$, b. $12x^2 -7$, c. $24x$, d. $12x-7$, e. 0

Correct Solution: b. $12x^2-7$

TABLE LXXI

Frequencies of Correct Answers, No Attempts & Errors Question 18

	N	%
Correct Answer	283	94.4
No Attempt	1	0.3
Gave answer as a. $12x^2$	0	0.0
Gave answer as b. $12x^2-7$	10	3.3
Gave answer as d. $12x-7$	5	1.7
Gave answer as e. 0	1	0.3

This question, involving a straight application of the formula for a differential coefficient, was exceptionally well done. However, the suggested answers should have contained an answer 8 for candidates who would have differentiated first without multiplying out the terms. 33% of the pupils failed to take the second differential coefficient as is indicated by response b. $12x^2-7$. Trouble with the index notation and indices in general may be advanced as a reason for response d. $12x-7$.

Question 19. $\int_4^9 \sqrt{x}\,dx$ is equal to a. 3 1/3, b. 1, c. 15 1/3, d. 19, e. 12 2/3

Correct Solution e. 12 2/3

TABLE LXXII

Frequencies of Correct Answers, No Attempts & Errors
Question 19

	N	%
Correct Answer	145	48.3
N_o Attempt	28	9.3
Gave answer as a. 3 1/3	32	10.7
Gave answer as b. 1	54	18.0
Gave answer as c. 15 1/3	21	7.0
Gave answer as d. 19	20	6.7

The ability to perform simple integration, by this group, is limited. Again, even though the process of integration, is understood, difficulty with exponents proves to be an insurmountable obstacle. The mechanics of the Arithmetic of small numbers is another factor which can account for wrong response. Those candidates who gave response b. 1 substituted for the $\sqrt{x}$ directly without integrating. These students no doubt understood that the integration had already been carried out for them. The example on p. 91 of the text is similar to this problem.

Question 20. The tenth and eleventh term of an Arithmetic Rrogression are 53 and 56 respectively. The fifth term is a. 26, b. 3, c. 35, d. 38, e. 41

Correct Solution: d. 38

TABLE LXXIII

Frequencies of Correct Answers, No Attempts, & Errors
Question 20

	N	%
Correct Answers	241	80.3
No Attempt	11	3.7
Gave answer as a. 26	15	5.0
Gave answer as b. 3	3	1.0
Gave answer as c. 35	18	6.0
Gave answer as e. 41	12	4.0

This question was very well done. Of significance is the fact that 11 candidates had no idea of how to work a simple problem in Arithmetic Progression. 5% of the candidates gave as their response a. 26 which is the first term of the series while 1% gave response b. 3 which is the common difference. 6% of the candidates found the fourth term of the series while 4% found the sixth term as is evidenced by responses c. 35 and e. 41, respectively. Example IV, p.100 of the text is similar to this problem.

Question 21. A point which lies on the graph of the equation $x^2-xy=21$ is a. (-3,5), b. (6,2), c. (3,-4), d. (-4,1), e. (5,1).

Correct Solution: c. (3,-4)

TABLE LXXIV

Frequencies of Correct Answers, No Attempts & Errors Question 21

	N	%
Correct Answer	216	72.0
No Attempt	40	13.3
Gave answer as a. (-13,5)	14	4.7
Gave answer as b. (6,2)	6	2.0
Gave answer as d. (-4,1)	14	4.7
Gave answer as e. (5,1)	10	3.3

On the whole responses to this question shows that some of the candidates have not a grasp of the fundamentals of analytical geometry. Problems similar to this one appear on page 72 of the Grade X Text. Again mechanical errors may be blamed for the incorrect responses which this group gave to this problem. Of particular significance is the fact that 13.3% of the students did not attempt this problem.

Question 22. In order that the function $y=9x-x$ shall be a maximum, the value of x must be a. 1, b. $\sqrt{3}$, c. 3, d. $-\sqrt{3}$, e. $\sqrt{9}$.

Correct Solution: b. $\sqrt{3}$.

TABLE LXXV

Frequencies of Correct Answers, No Attempt & Errors. Question 22

	N	%
Correct Answers	117	39.0
No Attempt	9	3.0
Gave answer as a. 1	58	19.3
Gave answer as c. 3	52	17.4
Gave answer as d. $-\sqrt{3}$	57	19.0
Gave answer as e. $\sqrt{9}$	6	2.0
Misread Problem	1	0.3

As results indicate this problem was very poorly done. Candidates showed little knowledge of the material covered on page 71 of the text. One candidate placed marks in two places on the provided sheet. This has been analyzed as the candidate having misread the problem. When the sheet was passed through the scoring machine, this question was marked wrong. 2% of the candidates failed to distinguish the similarity between response c. and e. A question similar to this one, dealing with cubic functions is Problem 2, Page 71 of the text.

Question 23. For positive integral values of the exponent n in the expansion $(a+b)^n$) the number of terms is a. n-2, b. n-1, c. n, d. n+1, e. n+2

Correct Solution: d. n+1

TABLE LXXVI

Frequencies of Correct Answers, No Attempt, Errors.
Question 23

	N	%
Correct Answers	142	47.3
No Attempt	25	8.3
Gave answer as a. n-2	5	1.7
Gave answer as b. n-1	54	18.0
Gave answer as c. n	58	19.3
Gave answer as e. n+2	14	4.7
Misread Problem	2	0.7

This problem was poorly done by students of this group. Their ability to generalize is limited as is evidenced by the results. Two candidates placed marks in more than one space on the sheet which was provided for the recording of responses. It is hard to visualize exactly how candidates arrived at the wrong solutions which they recorded.

Question 24. How may different three member executives consisting of a president, vice-president and secretary treasurer can be formed from a group of 10 persons?
a. 3, b. 120, c. 10^3, d. 72, e. 720

Correct Solution: e. 720

TABLE LXXVII

Frequencies of Correct Answers, No Attempts & Errors
Question 24

	N	%
Correct Answer	109	36.3
No Attempt	3	1.0
Gave answer as a. 3	0	0.0
Gave answer as b. 120	5	1.7
Gave answer as c. 10^3	2	0.7
Gave answer as d. 72	181	60.3

This problem was very poorly done, by pupils of this group. Example VI on p. 121 of the text is very similar. Surprising enough is the fact that only 7 candidates thought of this problem as one in combinations instead of permutations. Those who gave response c. 10^3 no doubt argued thus: $10C_1 \times 10C_1 \times 10C_1$. Those who gave response d. 72, arrived at the solution by dividing $10P_3$ by 10.

Question 25. If the first term of a geometric series is 5, the last term is 320 and the ratio is 2, then the number of terms is a. 5, b. 6, c. 7, d. 8, e.9.

Correct Solution: c. 7.

TABLE LXXVIII

Frequencies of Correct Answers, No Attempt & Errors.
Question 25

	N	%
Correct Answer	144	48.0
No Attempt	41	13.7
Gave answer as a. 5	23	7.7
Gave answer as b. 6	52	17.3
Gave answer as d. 8	23	7.7
Gave answer as e. 9	17	5.6

Results indicate a rather sketchy knowledge of the finding of the number of terms in a Geometric Series. As before pupils did not seem to have a sufficient knowledge of logarithms and exponents to be able to cope with this situation: $2^{n-1} = 2^6$. Those who gave a response of b. 6 in all probability expressed the n term as ar whence they would get $2^n = 2^6$ and $\therefore$ n = 6. There is no doubt but that the majority of students arrived at the correct solution by writing down the terms of the series from the given information.

Question 26. How many committees of three can be formed from a group of 12 men? a. 1320, b. 660, c, 440, d. 220, e. 110.

Correct Solution: d. 220

TABLE LXXIX

Frequencies of Correct Answers, No Attempts & Errors. Question 26

	N	%
Correct Answers	186	62.0
No Attempt	8	2.7
Gave answer as a. 1320	93	31.0
Gave answer as b. 660	7	2.3
Gave answer as c. 440	3	1.0
Gave answer as e. 110	3	1.0

This problem dealing with combinations was done much better than the problem dealing with permutations. 31% of the candidates treated this problem as one in Permutations while only 2.4% treated problem 24 as a problem in Combinations. Thus candidates could discriminate a problem in permutations but not one in Combinations. Again, candidates had difficulty in solving ${}^{12}C_3$, simply because of mechanical errors.

Question 27. The value of an automobile which sells for $2500 depreciates 20% in value each year. What is its worth at the end of four years? a. 0, b. $500, c. $1024, d. $1500, E. $1700.

Correct Solution: c. $1024

TABLE LXXX

Frequencies of Correct Answers, No Attempts and Errors Question 27

	N	%
Correct Answers	140	46.7
No Attempt	17	5.7
Gave answer as a. 0	7	2.3
Gave answer as b. $500	104	34.7
Gave answer as c. $1500	21	7.0
Gave answer as d. $1700	11	3.6

Results for this problem indicate a weak knowledge of successive discounts as taught in Grade VIII. It is significant to note that 17 candidates could not arrive at a solution to such a simple arithmetic problem. 34.7% of the students gave as their answer the depreciation after one year, b. $500. Responses c. $1500 and d. $1700 could well have been guess work on the part of the candidates.

Question 28. The coefficient a^3 in $(a+1)^7$ is

a. 3, b. 21, c. 30, d. 35, e. 42.

CORRECT SOLUTION: d.35

TABLE LXXXI

Frequencies of Correct Answers, No Attempts & Errors. Question 28

	N	%
Correct Answers	221	73.7
No Attempts	13	4.3
Gave answer as a. 3	16	5.3
Gave answer as b. 21	38	12.7
Gave answer as c. 30	7	2.3
Gave answer as e. 42	5	1.7

Results for this problem are quite good. However, the coefficients for a^3 and a^4 are equal in the expansion of $(a+1)^7$ and it may have been that students wrote down the formula for the coefficient of a^4 viz $\frac{n(n-1)(n-2)(n-3)}{4.3.2.1}$. Those who gave response b. 21 were finding the coefficient of a^2. Response e. 42 was given by those who failed to divide by 21. Response a. 3 and c. 30 are hard to justify. A problem similar to this one is problem 9 on p. 131 of the text.

Question 29. If $y = x^2$ what is the error in the approximation $\frac{\Delta y}{\Delta x} \simeq \frac{dy}{dx}$? a. Δx, b. Δx^2, c. $x\Delta x$, d. $2x\Delta x$, e. x.

CORRECT SOLUTION: b. Δx^2

TABLE LXXXII

Frequencies of Correct Answers, No Attempts & Errors Question 29

	N	%
Correct Answers	110	36.7
No Attempt	13	4.3
Gave answer as a. Δx^2	81	27.0
Gave answer as c. $x\Delta x$	10	3.3
Gave answer as d. $2x\Delta x$	82	27.3
Gave answer as e. x	4	1.4

As results indicate, this problem was poorly done. This problem is taken directly from the text where it appears as Example VIII on p. 79 . A complete solution is given for the problem. 27% of the candidates remembered the question from their texts where the error in approximation was given as $(\Delta x)^2$.

Question 30. Express the equation $x \log 5 = y \log 6 + 2$ in a form which does not involve logarithms.

a. $5x = 6y + 2$, b. $5^x = 100(6^y)$, C. $5^x = 6^y + 2$, d. $5^x = 6^y$ e. $5^x = 6^{y+2}$

Correct Solution: b. $5^x = 100\ (6^y)$

TABLE LXXXIII

Frequencies of Correct Answers, No Attempts & Errors Question 30

	N	%
Correct Answers	20	6.7
No Attempt	10	3.3
Gave answer as a. $5x = y + 2$	24	8.0
Gave answer as c. $5^x = 6^y + 2$	208	69.3
Gave answer as d. $5^x = 6^y$	5	1.7
Gave answer as e. $5^x = 6^{y+2}$	33	11.0

Results for this problem, as for problem, 16, show a decided lack of knowledge of logarithms. Very few candidates recognized the equation as a logarithmic one. The problem was taken directly from the Rivision exercises in the text; it appears as problem 11(VI) on p. 155 in Revision Exercise D. The knowledge of manipulation of logarithms, by pupils of this group, is slight.

Question 31. The sum to infinity of a geometrical series is 24 and the first term is 12. The correct series is: a. 12+4+1.33+... , b. 12+2+0.33+... c. 12+6+3+... ,d. 12+2.4+ 0.48+... ,e. 12+1.2+0.12...

Correct Solution: c. 12+6+3+...

TABLE LXXXIV

Frequencies of Correct Answers, No Attempts & Errors. Question 31

	N	%
Correct Answers	215	71.6
No Attempt	27	9.0
Gave answer as a.12+4+1.33+...	16	5.3
Gave answer as b.12+2+0.33+...	8	2.7
Gave answer as d.12+2.4+0.48+...	17	5.7
Gave answer as e.12+1.2+0.12+...	17	5.7

Results, for this problem were quite good. Candidates showed a knowledge of the sum to infinity of a Geometric Series.

Question 32. If $x^2-2xy+3y=4$, express y as a function of x. a. $\frac{x^2-4}{2x-3}$, b. $\frac{4-x^2}{2x+3}$, c. $\frac{4+x^2}{2x-3}$ d. $\frac{(x-2)(x+2)}{2x+3}$ e. $\frac{-x^2-4}{2x-3}$ Correct Solution: a. $\frac{x^2-4}{2x-3}$

TABLE LXXXV

Frequencies of Correct Answers, No Attempts & Errors Question 32

	N	%
Correct Answer	139	46.3
No Attempt	23	7.7
Gave answer as b. $\frac{4-x^2}{2x+3}$	74	24.7
Gave answer as c. $\frac{4+x^2}{2x-3}$	23	7.7
Gave answer as d. $\frac{(x-2)(x+2)}{2x+3}$	29	9.7
Gave answer as e. $\frac{-x^2-4}{2x-3}$	11	3.6
Misread Problem	1	0.3

The indicated results show a very poor knowledge of functional relationships. Considering that the pupil starts out in Grade IX to express a functional relationship, and has further work of this type in Grade X, the results are disappointingly poor. Problems such as this appear on p. 45 of the Grade X course and p. 222 of the Grade IX test. A faulty background in signed numbers can be given as a reason for the wrong responses. One candidate placed marks in more than one space on the provided sheet and as a result the problem was marked as wrong.

Question 33. The altitude (a) of a triangle is one-half the length of the base. Express the area of the triangle as a function of (a). a. $\sqrt{2a^2}$, b. $\sqrt{\frac{5}{4}a^2}$, c. a^2, d. $\frac{a^2}{4}$, e. 2a.

Correct Solution: c. a^2.

TABLE LXXXVI

Frequencies of Correct Answers, No Attempts & Errors Question 33

	N	%
Correct Answer	198	66.0
No Attempt	11	3.7
Gave answer as a. $\sqrt{2a^2}$	7	2.3
Gave answer as b. $\sqrt{\frac{5}{4}a^2}$	2	0.7
Gave answer as d. $\frac{a^2}{4}$	72	24.0
Gave answer as e. $2a^2$	10	3.3

This question was fairly well done. The ability to express a functional realtionship is fair only. Quite a few of the errors committed could no doubt be traced back to a language difficulty as is evidenced by response d. $\frac{a^2}{4}$. Candidates who gave this answer read the first part of the question as, " the base of a triangle is one- half the altitude."

Question 34. The perimeter of a square in feet and its area in square feet are expressed by the same number. Find the side of the square. a. 1 ft., b. 2 ft., c. 3 ft., d. 4 ft., e. 5 ft.

Correct Solution: d. 4 ft.

TABLE LXXXVII

Frequencies of Correct Answers, No Attempts and Errors
Question 34

	N	%
Correct Answer	238	79.3
No Attempt	11	3.7
Gave answer as a. 1 ft.	23	7.7
Gave answer as b. 2 ft.	20	6.6
Gave answer as c. 3 ft.	6	2.0
Gave answer as e. 5 ft.	2	0.7

This question was quite well done. It is hard to justify responses a., b., c. and e. outside of the obvious conclusion of straight guesswork.

B Questions 35-53.

The following tables show the errors which were made on the "Problem solving type", questions 35-53. In each case the problem will be given together with the correct solution as given by the Examinations Board. An effort has been made to classify each type of error made.

Question 35. Show that for positive values of r

$$\frac{r+2}{r+4} < \frac{r+4}{r+6}$$

Marking Scheme:

a. Finding lowest Common Denominator - 1 mark

b. $r^2+8r+12 < r^2+8r+16$ - 2 marks

c. $12 < 16$ - 1 mark

TABLE LXXXVIII

Frequencies of Correct Answers, No Attempts & Errors
Question 35

	N	%
Correct Answer	6	2.0
No Attempt	34	11.4
Substituted a numerical value for r	241	80.3
Error in Multiplication	1	0.3
Assumed $\frac{r+2}{r+4} = \frac{r+4}{r+6}$	6	2.0
Work was correct as far as step b. in solution(Partial marks awarded)	6	2.0
Attempted to graph $\frac{r+2}{r+4}$ and $\frac{r+4}{r+6}$ to show $\frac{r+2}{r+4} < \frac{r+4}{r+6}$	6	2.0

As the results indicate this problem dealing with ratio inequalities was very poorly done. 80.3% of the students substituted values for r directly, (anywhere from 0.01 to 1000), and arrived at the required conclusion. They were unable, however, to generalize directly from the ratio to show the inequality. 2% of the students attempted to graph the two ratios but with little success.

The following statement taken directly from a

paper indicates the muddled type of thinking which pupils of this group undergo when solving a problem of this type, "Since denominator always larger it is a larger fraction $\frac{4}{6} < \frac{6}{8}$ ".

The word numerator was spelled as "neumorator" on one paper.

Of significance here is the one paper on which the following appeared; $u_{n+1} > u_n$, $u_{n+1} < u_n$. This notation is used in Mathematics 55, a university course dealing with series and advanced calculus. The notation appears in the work on series. It appears then that at least one teacher has been demonstrating his knowledge of Math. 55 to his class. The work on series in Grade XII is so basic that the teaching of series in this manner is entirely unwarranted. It is entirely unnecessary to clutterup the pupils minds with these details of advanced courses. Pupils have a hard enough time of it to get the basic principles of Grade XII Mathematics.

Question 36. The safe load "W" (tons) for a beam varies jointly as the thickness "t" (inches) and the square of the depth "d" (inches) and inversely as the length "l" (feet) between the supports. If the safe load for a beam 6 inches thick, 10 inches deep and 12 feet long is 5 tons express W in terms of t,d, and l.

Correct Solution

1. $W = \frac{ktd^2}{l}$ - 1 mark
2. Substitution for W, t, d, l. - 1 mark
3. $k = l/10$ (answer must be correct - 2 marks

TABLE LXXXIX

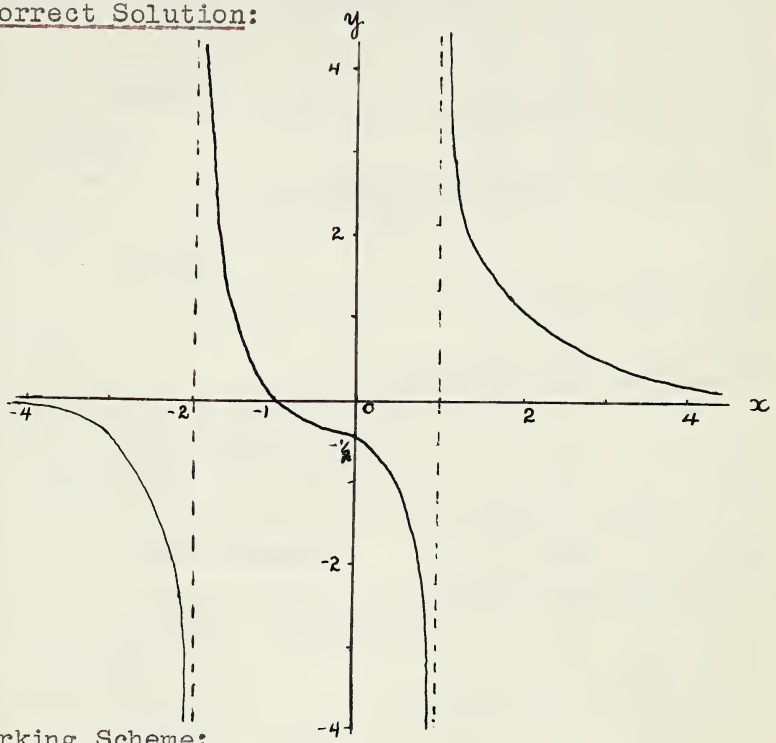
Frequencies of Correct Answers, No Attempts & Errors
Question 36

	N	%
Correct Answer	130	43.3
No Attempt	7	2.3
Solved for k in terms of W, t, d, l; did not substitute numerical value	11	3.7
Misread Problem e.g. square root for square	15	5.0
$W \propto \frac{1}{l^2}$ instead of $W \propto \frac{1}{l}$ - changed units to agree	63	21.0
Mechanical Errors e.g. 1. did not square "d" 2. Error in solution (got k=10) 3. $10^2=1000$ 4. Multiplication	26	8.7
Correct Solution - Error on markers part in not awarding total marks.	2	0.7
Miscellaneous e.g. 1. Did not reduce result. 2. Translation Error.	13	4.3
Inability to solve for k i.e. did not include the proportionality constant in the relation.	33	11.0

Of the 19 "problem solving" type problems on the paper, question 36 received more correct responses than any other question of this type. This problem is similar to example VII on p. 17 of the text. Solutions to this question indicate clearly that 34.3% of the students of this group had not the ability to express the information of the problem in joint variation form. 18% of the students were unable to arrive at a correct solution because of mechanical errors of one type or another. As results indicate, pupils of this group, found this problem relatively difficult.

Question 37. Using the axes below, sketch the graph of the function $\frac{x+1}{(x-1)(x+2)}$

Correct Solution:



Marking Scheme:

a. Asymptotes at x=1 and -2 ($\frac{1}{2}$ mark each) - 1 mark

b. Behavior of graph near asymptotes -2 marks ($\frac{1}{2}$ mark each

c. Intersection of graph with (x axis-1 mark ($\frac{1}{2}$ mark each)
(y axis

d. Behavior of graph for large positive and negative values of x - 1 mark ($\frac{1}{2}$ mark each)

e. If sketch is reasonably accurate - 1 mark

TABLE XC

Frequencies of Correct Answers, No Attempts & Errors
Question 37

	N	%
Correct Answers	92	30.7
No Attempts	3	1.0
Placed extra branches of the function in the (-x,y) and (x,-y) guadrants	7	2.3
Analyzed the function as a parabola	38	12.7
Inverted middle section of function i.e. between the vertical asymptotes or could not sketch it.	56	18.6
Left out side branches	24	8.0
Confused attempt - impossible to state thinking undergone in analysis	38	12.7
Put in horizontal asymptote at y=1 and could not sketch the central section of the function	6	2.0
Sketch not accurate	36	12.0

Results indicate a very weak knowledge of graph sketching. The main weakness is in analysis of the function. Difficulty with signs may be advanced as a reason why 2.3% of the candidates misplaced the side branches of the function.

Problems similar to this appear on p. 28 of the Text. Pupils are given an introduction to graph sketching in Grade X. Without a doubt, sketching of functions should be taught thoroughly and a great deal of practise should be given. The number of examples in the text should be supplemented.

Question 38. On the axes provided below, sketch a graph which has the following characteristics:

a. $\longrightarrow 2$ when $x \longrightarrow + \infty$

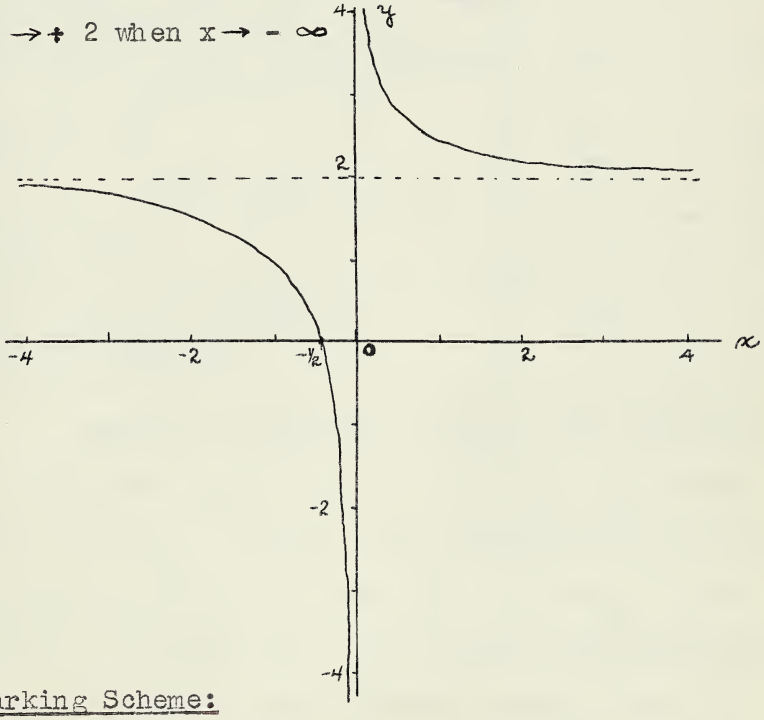
b. $\longrightarrow + \infty$ when $x \longrightarrow 0$ provided $x > 0$

c. Is never equal to + 2

d. $\longrightarrow - \infty$ when $x \longrightarrow 0$ provided $x < 0$

e. Is equal to zero when $x = -\frac{1}{2}$

f. $\longrightarrow + 2$ when $x \longrightarrow - \infty$



Marking Scheme:

a. One half mark for each part a, b.,c.,d.,e., and f. above.

Frequencies of Correct Answers, No Attempts & Errors
Question 38

	N	%
Correct Answers	109	36.3
No Attempt	34	11.4
Confused attempt - impossible to state the thinking candidates underwent in sketching	54	18.0
Right Branch incorrect e.g. approached x axis asymptotically	31	10.3
Analysis attempted as for question 37	40	13.3
Vertical asymptote at $x = 2$	16	5.4
Rough sketch	4	1.3
Left Branch incorrect e.g. placed completely in $(-x,-y)$ guadrant	12	4.0

From the indicated results, this problem was poorly done. Question 13 of the text is similar to this problem. 13.3% of the students had the function of problem 37 before them, and the sketch which they made was very similar to the sketch in problem 37. A great number of the students tried to set up the function and as a result lost a great deal of time in doing so. Again, the inability to read the notation and interpret it correctly resulted in many

incorrect sketches. Although the marking scheme makes no mention of loss of marks for a rough sketch, 1.3% of the pupils lost 1 mark for this.

Question 39. An open cardboard box with square ends is fitted with an overlapping lid which covers the other side. The total area of the cardboard in box and cover is 8 square feet. That is the length of the square end so that the volume of the box will be a maximum?

Correct Solution

1. $4x^2+5xy=8$, $y=\frac{8-4x^2}{5x}$ - 2 marks (1 mark each)
2. $V = x^2(\frac{8-4x^2}{5x})$ or $\frac{8x-4x^3}{5}$ - 1 mark
3. $\frac{dV}{dx} = \frac{8-12x^2}{5}$ - 1 mark
4. Some proof to show that $x = \sqrt{\frac{2}{3}}$ is the value which gives a maximum - 1 mark.
5. Correct answer - Length of square edge $= \sqrt{\frac{2}{3}}$ or 0.82 or 0.816 ft. - 1 mark

TABLE XCII

Frequencies of Correct Answers, No Attempts & Errors
Question 39

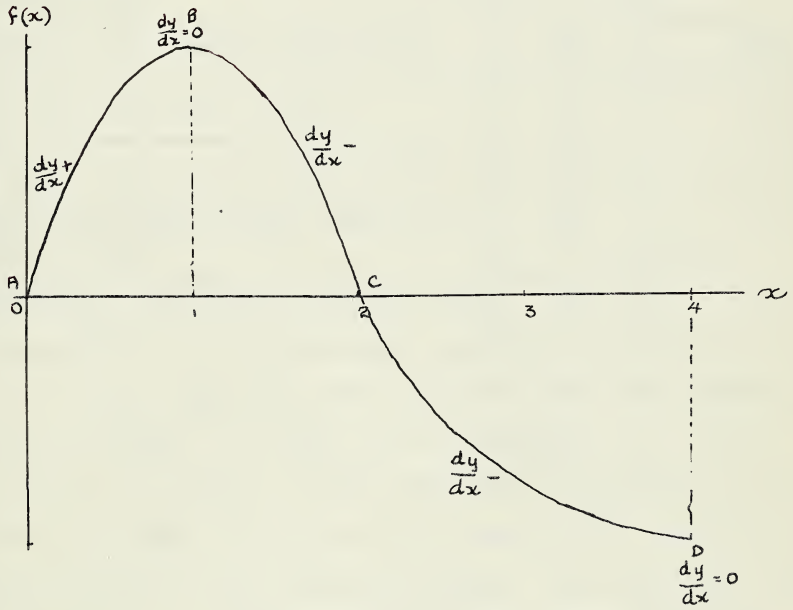
	N	%
Correct Answer	2	0.7
No Attempt	28	9.3
Found volume and could go no further in solution - 3 marks	4	1.3
No proof for maximum value and no conclusion as to numerical value of width - 4 marks	7	2.3
Used too many unknowns e.g. 1. l,w,h. 5 2. x+l,l,y 2	7	2.3
Preliminaries only e.g. drew figure, assigned unknowns, doodled, etc.	62	20.7
Wrong analysis: 1. Found Perimeter 1 2. Put l = w = h 33 3. 6 sides instead of 5 16 4. 4 sides instead of 4 17 5. 3 ends instead of 4 4 6. 2 ends instead of 4 14	85	28.3
Solved correctly for length in terms of the area; could go no further	52	17.3
Translation Error	5	1.7
Mechanical Errors - Differentiation indices, etd.	26	8.7
Should have had more marks	19	6.4
Miscellaneous 1. "Length must be o.64 1 2. Psychic guess as to dimensions 1 3. Copied result-work did not point to result obtained 1	3	1.0

As results indicate, problems of this type are much too subtle for pupils of this group. The two perfect results were achieved by two pupils whose transmuted score was just 50%. It is hard to envisage how students in this category could think through a problem such as this. On closer examination of the two papers, it was found that on one paper only two other questions were correctly done viz. # 48 and 51; while on the second paper # 36, 37, 47 were correctly do 28.3% through inability or not reading the question closely enough analyzed the problem incorrectly. The differentiation errors were mainly of the following type $\frac{d}{dx}\left(\frac{8x-4x^3}{6}\right) = 8-12x^2$, $\frac{d}{dx}(6x.8x^{-1}) = 6x-8x^{-2}$. Problems of this type are found on p. 77 of the text. Of particular significance here is the fact that markers overlooked 6.4% of the cases where more marks should have been given.

Question 40. Sketch on the axes below, a graph of a function from x=o to x=4 satisfying the data in the following table:

X	0	0-1	1	1to2	2to4	4
y	0			+	-	
$\frac{dy}{dx}$	+	+	0	-	-	0

Correct Solution



Marking Scheme:

1. One mark for each portion of curve AB, BC, CD - 3 marks.
2. One mark for positive slope at A - 1 mark.
3. Zero slope at B and D - 1 mark ($\frac{1}{2}$ mark each).

TABLE XCII

Frequencies of Correct Answers, No Attempt & Errors.
Question 40

	N	%
Correct Answer	85	28.3
No Attempt	59	19.7
Confused Attempt-Impossible to state why candidate sketched what he did.	68	22.7
Did not show zero slope at D	45	15.0
Did not show x=o at y= o	9	3.0
Did not show zero slope at=B	12	4.0
Failed to show graph cutting x axis at x=2	12	4.0
Showed $\frac{dy}{dx}$ negative in region AB	10	3.3

This question was difficult for this group. Pupils showed, as they did in question 37, little ability to sketch a graph when the information was given to them. It is hard to justify the sketches which 22.7% of the students made. The sketches showed absolutely no relationship to the data which was supplied. Problems similar to this one appear on P. 72 and 73 of the text. It is reasonable to say that pupils of this group were unable to establish the relationship between $\frac{dy}{dx}$ being + or - and the curve itself.

Question 41. Find the value of $\frac{8132h^{2.5}}{n^2}$ when h= 4.65 and n=325, correct to two decimals.

Correct Solution:

1. log 8132 + 2.5 log 4.65 - 2 log 325 - 2 marks
2. 3.9102 + 1.6688-5.0238 - 1 mark
3. Anti log 0.5552 = 3.59 - 1 mark

TABLE XCIV

Frequencies of Correct Answers, No Attempts & Errors.
Question 41

	N	%
Correct Answer	62	20.6
No Attempt	14	4.7
Substitution made - no other work attempted	31	10.3
Error in indices e.g. $(8132x4.65)^{2.5}$ inability to evaluate $(4.65)^{2.5}$, squared 4.65, denominator was not squared.	32	10.7
Translation Error e.g. $h^{2.5} = h^7$ or h^{10}	26	8.7
Mechanical Errors 1. Multiplication 19 2. Addition & Subtraction 5	24	8.0
Significant figures - did not express answer correct to two decimal places.	30	10.0
Did not use logs to evaluate-attempted to multiply	25	8.3
Multiplied log 4.65 by log 8132 and divided by log 325^2	6	2.0
Error in finding logs	41	13.7
Error in finding anti-log	9	3.0

As results indicate, pupils of Grade XII, in this category, show very little ability in handling logarithms. This point is further borne out by the showing which this group made on questions 16 and 30. It is hard to envisage how students who are going to continue with further academic training can hope for much success in the field of Mathematics. It is rather startling to think that 8.3% of the candidates attempted to evaluate $h^{2.5}$ by long hand multiplication. The mechanical errors made were mainly in multiplication addition and subtraction of decimals. 16.7% of the candidates experienced difficulty in finding the logs and anti logs of numbers, possibly because of insufficient practise. This question was taken directly from the text, where it appears as question 8 of Revision Exercise K, page 177.

Question 42. Given $y = f(x)$

a. State in words the meaning of Δy.

b. For a curve, state in words the geometrical interpretation of $\frac{\Delta y}{\Delta x}$.

c. For a curve, state in words the meaning of

$\frac{dy}{dx}$.

Correct Solution:

a. For a change Δx in x, Δy is the corresponding change in y. - 2 marks.

b. For two points on the curve, $\frac{\Delta y}{\Delta x}$ is the slope of the line joining these two points - 1 mark.

c. For a point on the curve, $\frac{dy}{dx}$ is the slope of the tangent at that point - 2 marks.

TABLE XCV a

Frequencies of Correct Answers, No Attempts and Errors
Question 42a

	N	%
Correct Answer	63	21.0
No Attempt	21	7.0
Δy means the change in the function	64	21.4
A small increase in y	47	15.7
Average gradient or rise	18	6.0
$\Delta y = \frac{dy}{dx}\Delta x$ 14 Δy is the error in taking $\frac{dx}{dy}$ 4	18	6.0
Addition of Curve	4	1.3
Change in y with respect to x	10	3.3
$\Delta y = f(x+\Delta x) - f(x)$	27	9.0
Rate of change of slope	8	2.7
$\Delta y = \Delta x$	6	2.0
Exact value of the function	3	1.0
Small portion of the graph between y and Δy.	6	2.0
Increase in volume	1	0.3
Δy is the tangent	4	1.3

Candidate showed, on the whole, little ability to express a generalization. 9% of the candidates knew how to go about finding Δy but were unable to express this relationship in words. As results indicate there is a great deal of confused thinking by students of this group in regard to the whole topic of differentiation.. However, they are able to apply the formula for the differential coefficient with some ease.

TABLE XCV b

Frequencies of Correct Answers, No Attempts & Errors
Question 42 b

	N	%
Correct Answer	116	38.7
No Attempt	27	9.0
Change in the variable over the change in the function 19	22	7.3
$\frac{f(x+\Delta x) - f(x)}{\Delta x}$	9	3.0
Increase in y of the curve at a point in realtion to x	18	6.0
Tangent at a certain point	9	3.0
Error on the y axis divided by error of the area along the x axis	3	1.0
Rise over the run	9	3.0
$\frac{\text{delta y}}{\text{delta x}}$	4	1.3
Gradient or slope of the curve 35 Gradient of tangent 11 Gradient at a point 7 Slope of the secant 3 Gradient of the increase of the curve 8	64	21.3
Miscellaneous 1. Secant of the curve 6 2. Height of triangle to its base 2 3. Geometrical interpretation is integration 3 4. The part greater than $\frac{Y}{x}$ by $\frac{\Delta y}{\Delta x}$ 2 5. Average rate of change over a period 6	19	6.4

Again, candidates experienced difficulty in expressing a relationship. 30% knew how to do the mechanics of finding $\frac{\Delta y}{\Delta x}$ but were unable to state this in words. Other responses indicate fairly accurately the amount of knowledge possessed by this group on this topic.

TABLE XCV c

Frequencies of Correct Answers, No Attempts & Errors.
Question 42c

		N	%
Correct Answer		99	33.0
No Attempt		21	7.0
$\frac{dy}{dx}$ gives slope of curve	8		
Rate of change of slope of curve.	12		
Slppe of tangent at a point.	10		
Average gradient of the curve.	29		
Point where gradient=o	4		
Slope of curve at apoint	10	73	24.4
Miscellaneous			
1. Velocity of speed	6		
2. Secant of the curve	8		
3. Area under curve	1		
4. Tells you when curve crosses x axis	1		
5. Tangent to a slope	1	17	5.7
Turning point of a curve when $\frac{dy}{dx}$=o, gives the tangent		16	5.3
Average rate of increase at a point		4	1.3

cont'd.

TABLE XCV c-concluded

$\frac{dy}{dx}$ is the derivative i.e. the function with one less and the number in front multiplied by the power of x in the function	4	1.3
$\frac{dy}{dx}$ is tangent of the curve	5	1.7
Average tangent of the curve	3	1.0
Curve at a special place on the curve	1	0.3
$\frac{dy}{dx} = \frac{\Delta y}{\Delta x}$	15	5.0
Average rate of change at a point	8	2.7
Gradient of a tangent	16	5.3
$\frac{dy}{dx} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$	5	1.7
Differential of y with respect to x	13	4.3

This question was done much better by the students of this group. 27% of the candidates received 1 or 2 marks for the a. part while 53% received 1, or 2 marks for the c. part. On the whole this question was relatively difficult for these students. The topic on differentiation remains to them as something which is quite nebulous. 24.4% of the candidates had the idea that $\frac{dy}{dx}$ represented a slope of some kind or other. 1.7% of the students showed some knowledge of how $\frac{dy}{dx}$ was arrived at from first principles. Not one paper however, mentioned limit in their definitions.

Question 43. Given $y = \frac{x^2 + 1}{\sqrt{x}}$

Find $\frac{dy}{dx}$ when $x = 4$

Correct Solution:

$\frac{dy}{dx} = \frac{d}{dx}(x^{\frac{3}{2}}) + \frac{d}{dx}(x^{-1/2})$ or $y = x^{3/2} + x^{-1/2}$ - 1 mark

$\frac{dy}{dx} = \frac{3x^{1/2}}{2} - \frac{1}{2}x^{-3/2}$ - 2 marks (one mark each)

For $x = 4$, $\frac{dy}{dx} = 2\frac{15}{16}$ - 2 marks

TABLE XCVI

Frequencies of Correct Answers, No Attempts & Errors
Question 43

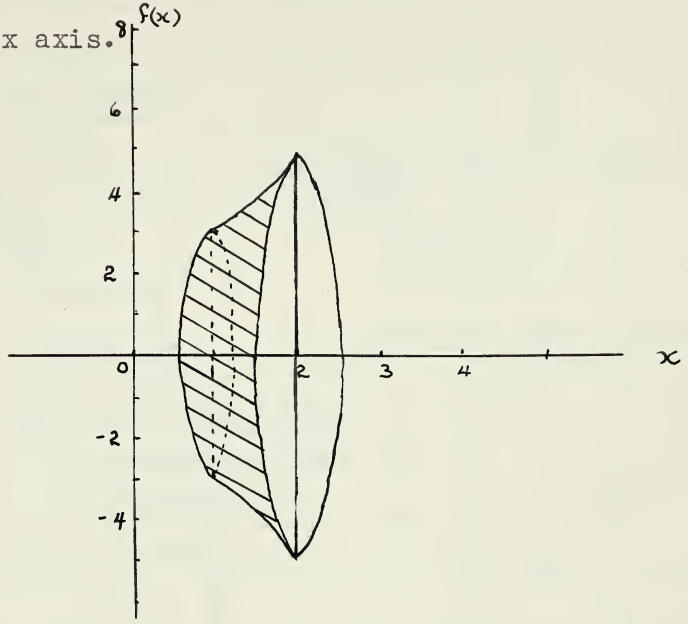
	N	%
Correct Answer	56	18.7
No Attempt	2	0.7
Mechanical Errors in Division 1. $\frac{x^2+1}{\sqrt{x}} = x^2+1+x^{-1/2}$ 71 2. $\frac{x^2+1}{\sqrt{x}} = x^2+x^{-1/2}$ 29 3. $\frac{x^2+1}{\sqrt{x}} = x^{3/2}+1$ 3	103	34.3
Errors in Differentiation 1. $\frac{d}{dx}(x^{-1/2}) = x^{-3/2}$ 4 2. $\frac{d}{dx}(\frac{x^2+1}{\sqrt{x}}) = (2x)(x^{-1/2})$ 17 3. $\frac{d}{dx}(\frac{x^2+1}{\sqrt{x}}) = \frac{2x}{1/2\sqrt{x^3}}$ 36 4. $\frac{d}{dx}(x^{3/2}) = \frac{3}{2}x^{5/2}$ 4	61	20.3

Question 43-concluded

Errors in Indices 1. $\frac{1}{\sqrt{x}} = x^{\frac{1}{2}}$ 7 2. $\frac{1}{2}x^{-3/2} = \frac{1}{2}\sqrt[3]{4}$ 1 3. $\frac{1}{\sqrt{x}} = x^{-1}$ 6 4. $3/2\ x^{\frac{1}{2}} = \frac{3}{2\sqrt{x}}$ 3 5. $x^{-1/2}(x^2+1) = x^{-1}+x^{-1/2}$ 3 6. $(x^{-1/2})(x^2) = x^{-1/4}$ 3 7. $3/2\ x^{1/2} = \sqrt{1.5x}$ 2	25	8.3
Did not substitute x=4 to find value of $\frac{dy}{dx}$	5	1.7
Substituted directly in $\frac{x^2+1}{\sqrt{x}}$ without differentiating first	21	7.0
Translation Error	16	5.3
Other mechanical Errors 1. $\sqrt{4^3}$ 4 2. $2-\frac{1}{2}=1$ 3 3. $3/2\sqrt{4} = 6/4$ 3	10	3.4
Should have had full marks	1	0.3

From the indicated results, it is clear that this problem was handled poorly by candidates in this group. It is evident that fundamentals have not been thoroughly enough absorbed. 66% of the candidates were undable to achieve a solution because of a limited knowledge of the fundamentals required for this grade. Certainly no case can be built up for the 7% who did not recognize the problem as one in simple differentiation. Surprising enough is the fact that only 2 candidates of this group did not attempt this problem. Four candidates integrated rather than differentiated. Problems 6 and 7 on p. 168 of the text are similar to this one.

Question 44. The figure below represents a solid formed by rotating part of the graph of x^2+1 about the x axis.



Find the volume to the nearest whole number.

Correct Solution:

1. $\pi\int_1^2 (x^2+1)^2\,dx$ or $\pi\int_1^2 (x^4+2x^2+1)\,dx$ - 1 mark

2. $\pi\left[\frac{x^5}{5}+\frac{2x^3}{3}+x\right]_1^2$ - 2 marks

3. $\pi\ 178/15$ - 2 marks

4. 37 cu. units - 1 mark

TABLE XCVII

Frequencies of Correct Answers, No Attempts & Errors
Question 44

		N	%
Correct Answers		20	6.7
No Attempt		21	7.0
Error in Formula for finding volume.			
1. $\pi \int^{2} (x^2+1)\, dx$	48		
2. $\pi \int^{2} (x^2+1)^3\, dx$	4		
3. $V = \frac{\pi}{3} \int^{2} (x^2+1)^2\, dx$	2	54	18.0
Mechanical Errors			
1. Addition & Subtaction	9		
2. Multiplication & Division	21		
3. Integration Errors e.g. $\int x^4 = \frac{x^5}{4}$	15		
4. Error in Evaluation e.g. $\int x^4 = \frac{x^5}{4}$ $\frac{2}{3}x^2 = \frac{40}{5}$ when $x = 2$	13	58	19.4
Did not multiply final result by π		22	7.3
Translation Error		13	4.3
Did not integrate-substituted directly		8	2.7
Limits			
1. Wrong Limits	56		
2. Could not find limits	5	61	20.3
Differentiated		21	7.0
Did not bring result to nearest whole number		7	2.3
Did not evaluate at upper or lower limit		4	1.3
Integrated before squaring		3	1.0
Should have had more marks		3	1.0
Incomplete-right as far as pupil went		5	1.7

The indicated results show clearly that pupils found this problem difficult, only 30% of the pupils were able to secure 50% or more of the marks given for this problem. Mechanical errors again took a heavy toll. 20% of the candidates had difficulty in evaluation, Integration and the arithmetic of small numbers. 10% of the candidates were not aware of the problem for they either differentiated or substituted directly into the formula. Inability to read a problem carefully proved costly for 2.3% of the candidates. 7.3% of the candidates placed π in their formula, but forgot to multiply the final result by it. Inability to state the limits of integration earned a zero score for 20.3% of the candidates. No marks were given unless the limits were correct. The upper limit ranged anywhere from 1 to 5 while the range of the lower limit was 0 to 5. Problems similar to this one appear on p. 92 of the text.

Question 45. Solve the equation $5+\sqrt{x-2} = 2x$

Correct Solution

1. $x-2 = (2x-5)^2$ - 1 mark.
2. $4x^2-21x+27=0$ - 1 mark
3. $x = 3$ or $9/4$ - 2 marks
4. $9/4$ is an extraneous root - 1 mark

TABLE XCVIII

Frequencies of Correct Answers, No Attempt & Errors
Question 45

Correct Answer	5	1.7
No Attempt	12	4.0
Error in $\sqrt{b^2-4ac}$ e.g. $-4ac=432 \neq 108$	5	1.6
Did not transpose -squared equation as given e.g. $5+\sqrt{x-2}=25+x-2$	121	40.3
9/4 was not listed as extraneous-received partial marks	32	10.7
Attempted to use the formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ directly	15	5.0
Mechanical Errors in multiplication, division, addition and subtraction	41	13.7
Attempted to differentiate 13 Attempted to integrate 2 Attempted to expand $(x-2)^{1/2}$ by binomial expansion 1	16	5.3

cont'd.

TABLE XCVIII
concluded..

Not complete - started correctly	21	7.0
Translation Error	8	2.7
Guessed at oneof the roots	3	1.0
Got as far as step 2 in solution-could not factor the equation	6	2.0
Expressed x as a function of x	4	1.3
Squared (x-2)	3	1.0
Squared $(5+\sqrt{x-2})$; then could not factor	8	2.7

As results indicate, this question was much too difficult. Only 18% of the candidates were able to secure a mark of 2 or better. 40% of the candidates of this group squared $5+\sqrt{x-2}$ as indicated. Certainly after having practise in squaring in Grade X, one would not expect this. Those who attempted to differentiate or integrate or expand $(x-2)^{1/2}$ by the Binomial Expansion had no insight into what the problem asked. 2% of the students could not apply the formula for the solution of a quadratic equation. This same problem appears on p. 150 of the text where it is question 10.

Question 46. The first term of a Geometric Progression is 1, the last term is 512 and the common ratio is 2. Find the number of terms.

Correct Solution:

1. $ar^{n-1} = 512$ - 1 mark

2. $2^{n-1} = 512$ - 1 mark

3. $n = 10$ - 1 mark

1 mark only for $n = 10$ by writing down the 10 numbers.

TABLE XCIX

Frequencies of Correct Answers, No Attempts & Errors.

	N	%
Correct Answers	64	21.3
No Attempt	33	11.0
Attempted to use the formula for the sum of a Geometric Series	36	12.0
Wrote down the terms- wrong result	5	1.7
Mechanical Errors	12	4.0
Could not solve $2^{n-1} = 512$	32	10.7
Wrote down terms- correct result	38	12.7
Treated the question as one in Arithmetic Progression	18	6.0
Correct start - incomplete	12	4.0
Should have had more marks	8	2.7
Confused Attempt e.g. 1. n^{th} term $= 2^{511}$ 2. $512 = 2^{511}$	27	9.0
$2^n = 512$	7	2.3
Translation Error	7	2.3
Attempted to Use Binomial Expansion.	1	0.3

Results for this question were better than for most of the questions in this part of the paper. It is interesting to note that questions 25 and 46 are similar. 48% of the candidates received perfect marks for question 25 while only 21.3% of the candidates received perfect marks for question 46. Again, the inability to use logarithmic manipulation proved costly for 10.7% of the students. Those who attempted to use the formula for the sum of a G. P. or treated the problem as one in A. P. had no insight into the problem.

Question 47. For each birthday, a daughter receives $1.00 for every year of her age. How much has she received in all when she reaches her twenty-first birthday?

Correct Solution:

1. a=1; d=1; n=21; = 1 mark
2. Summation - 1 mark
3. Amount = $231 - 1 mark

1 mark only for arriving at answer by writing down the terms.

TABLE C

Frequencies of Correct Answers, No Attempts & Errors
Question 47

	N	%
Correct Answers	106	35.3
No Attempts	5	1.6
Wrote down terms-correct results	35	11.7
Correct Start-Incomplete	9	3.0
Wrote down terms-incorrect results	26	8.7
Wrong analysis of problem: $1 x 21 = $21	57	19.0
Mechanical Errors	14	4.7
Attempted to find sum of series by applying the formula for the n^{th} terms of an A.P.	8	2.7
Took n = 2 instead of 21	1	0.3
Wrong analysis: $(1.00)^{21}$ = $1.00	1	0.3
Summed as a G.P.	5	1.7
Took n = 20 instead of 21	17	5.7
Confused attempt e.g. 1. 21 ! 2. $^{1}C_{1} + {}^{1}C_{2} + \ldots + {}^{1}C_{21}$	15	5.0
Should have had more marks	1	0.3

On the whole the results for this problem showed that candidates had some insight into what was required. Again in a problem as simple as this one, mechanical errors were a stumbling block to success. Those who treated the problem as one in G. P.-2% and those who applied the formula for the n^{th} term of an A. P. had little insight into the problem.

Question 48. An annuity of $100 beginning now (first payment due one year from now) has 15 years to run. It is left uncollected by the payee and accumulates at the rate of 2% compounded annually. Find the value of the annuity at maturity.

Correct Solution:

1. $100 + 100\,(1.02) + 100\,(1.02)^2 + \ldots + (1.02)^{14}$ -2 marks

2. $100\left\{\frac{(1.02)^{15}-1}{1.02-1}\right\}$ 1 mark

3. Amount = $1729.35 1 mark

TABLE CI

Frequencies of Correct Answers, No Attempts & Errors
Question 48

	N	%
Correct Answers	98	32.7
No Attempt	23	7.7
Misread Problem-worked as a problem in Present Worth	45	15.0
Expressed answer to the nearest dollar.	8	2.7
Summed to 14 or 16 terms only	15	5.0
Error in setting down series	15	5.0
Not summed as a G. P.- read problem as one in compound interest	42	14.0
Mechanical Errors	20	6.6
Confused Attempt	8	2.7
Error in applying formula for the sum of a Geometric Series	10	3.3
Translation Error	3	1.0
Incomplete	11	3.6
Should have had more marks	2	0.7

Candidates found this problem difficult. 29% of the pupils recognized the problem as being from the chapter on sequence and Series and worked the problem either as one in Present Worth or Compound Interest. Problem 3 on p. 116 of the text is similar.

Question 49. Eight flags, three of which are white, three red and two green are all displayed in a column. How many different signals may be shown?

Correct Solution:

1. $\frac{8!}{3!\ 3!\ 2!}$ -2 marks

2. 560 signals -1 mark

TABLE CII

Frequencies of Correct Answers, No Attempts & Errors
Question 49

	N	%
Correct Answer	3	1.0
No Attempt	10	3.3
${}^5C_3 \times {}^5C_3 \times {}^6C_2$	6	2.0
${}^8P_3 + {}^5P_3 + {}^2P_2$	7	2.3
Drew flags-doodled, etc.	22	7.3
${}^6C_3 + {}^6C_3 + {}^7C_2$	2	0.7
${}^8C_3 + {}^8C_3 + {}^8C_2$	56	18.6
${}^8C_3$	18	6.0
${}^3P_3 + {}^3P_3 + {}^3P_2$	11	3.7
${}^8P_3 + {}^8P_3 + {}^8P_2$	37	12.3
8!	69	23.0
${}^8C_3 \times {}^5C_3$	2	0.7
${}^8C_3 + {}^8C_2 + {}^8C_1$	5	1.7
${}^8C_3 + {}^5C_3 + {}^5C_2$	12	4.0
8+7+6+5+4+3+2+1	2	0.7
${}^3C_1 \times {}^3C_1 \times {}^2C_1$	11	3.7
${}^6P_3 \times {}^6P_2$	5	1.7

cont'd.

TABLE CII
Concluded

$3! \times 3! \times 2!$	7	2.3
${}^8P_5 + {}^8P_5 + {}^8P_6$	1	0.3
3^8	2	0.7
$8^3 + 7^3 + 6^2$	2	0.7
${}^8C_2 \times {}^8C_3 \times {}^8C_3$	10	3.3

This problem was much too difficult for the students in this group. 1% received full marks for it while 99% received a mark of zero. All responses which pupils may have been tabulated as in Table XLIX.

Question 50. Show by using the Binomial Theorem that the cube root of 28 is approximately 3.037.

Correct Solution:

1. $(3^3)^{1/3}\left\{1+1/3\right)^{1/3} = (3+\frac{1}{27})^{1/3}$ 2 marks
2. $3(1+\frac{1}{3}\cdot\frac{1}{3^3}+\ldots.) = 3(1+\frac{1}{81}+\ldots.)$ 1 mark
3. 3.037 - 1 mark

TABLE CIII

Frequencies of Correct Answers, No Attempts & Errors.
Question 50

	N	%
Correct Answers	50	16.7
No Attempt	51	17.0
Used the approximation formula $(1+x)^n \simeq 1+nx$ for $(3+0.37)$	19	6.3
Attempted to expand by Binomial Theorem $(27+1)^{1/3}$	46	15.3
Mechanical Errors	26	8.7
$\sqrt[3]{28} = 3.037$ or $(3.037)^3 = 28$	33	11.0
Error in evaluation of second term of expansion	5	1.7
$10\ (2+.8)^{1/3}$	5	1.7
$\sqrt[3]{30\ -2}$	11	3.6
$\sqrt[3]{25+3}$	5	1.7
Doodled, rewrote information etc.	33	11.0
Incomplete	12	4.0
Used logs to evaluate and ran into difficulty of log manipulation	4	1.3

As results indicate, this question was difficult for candidates in this group. 22% of the candidates showed little knowledge of the application of the Binomial approximation. Mechanical errors in multiplication resulted in 11% of the pupils receiving a zero score. Problems similar to this one appear on p. 132 of the text.

Question 51. Given $Y = 4x - x^3$

Find the maximum value of Y correct to two decimal places.

Correct solution:

1. $\frac{dy}{dx} = 4 - 3x^2$ - 1 mark

2. $4 - 3x^2 = 0$ - 1 mark

3. $x = \pm \frac{2}{\sqrt{3}}$ or $\pm$ 1.15 - 1 mark

4. Some proof to show that at $x = \pm \frac{2}{\sqrt{3}}$, function has a maximum value - 1 mark.

5. 3.08 ans - 1 mark

TABLE CIV

Frequencies of Correct Answers, No Attempts & Errors
Question 51

	N	%
Correct Answers	12	4.0
No Attempts	14	4.7
Mechanical Errors	58	19.3
Translation Errors	6	2.0
No proof given for maximum value found	37	12.3
Maximum value was not found	77	25.7
Graphed function and attempted to find the maximum value from the graph.	18	6.0
Wrong hypothesis in proof for the maximum value	8	2.7
Error in differentiation e. g. mixed differentiation with integration	20	6.7
Should have had more marks	9	3.0
Guessed at maximum value	4	1.3
Did not evaluate for y	37	12.3

From the indicated results, candidates of this group experienced a great deal of difficulty with this problem. Little difficulty was experienced in differentiating the function. Candidates however, showed little knowledge of what to do once they had differentiated. Mechanical and translation errors accounted for 21% of the incorrect responses.

Question 52.

Find the area bounded by the curve

$y = \sqrt{x}\ (4 - x)$ and the x axis.

Correct Solution:

1. $\int_0^4 \sqrt{x}\ (4 - x)\ dx$ - 2 marks
2. $\left[2/3\ x^{3/2} - 2/5\ x^{5/2} \right]_0^4$ - 1 mark
3. Substitution - 1 mark.
4. Ans 8 8/15 or 8.53 sq. units - 1 mark

TABLE CV

Frequencies of Correct Answers, No Attempts & Errors
QUESTION 52

	N	%
Correct Answers	4	1.3
No Attempts	52	17.3
Integration Errors e. G. $\int x^{1/2}(4-x)dx = \frac{2x^{3/2}}{3}(4x-\frac{x^2}{2})$	49	16.3
Mechanical Errors	13	4.4
Doodled, drew sketch etc.	59	19.7
Could not find limits 67 Wrong limits 19	86	28.7
Found volume instead of area	19	6.3
Translation Error	5	1.7
Differentiated instead of integrated	9	3.0
Should have had more marks	1	0.3
Did not Integrate - direct substitution	3	1.0

This question was much too difficult for students of this group; 83.4% of the candidates received a score of o. The major stumbling blocks in this question seemed to be the inability to find the limits of integration and secondly, to integrate the function. 10.3% of the candidates had no insight into the problem for they differentiated, found the volume or substituted directly into the function. Strangely enough those who substituted directly, had the right limits. A problem similar to this one is found on p. 93 of the text.

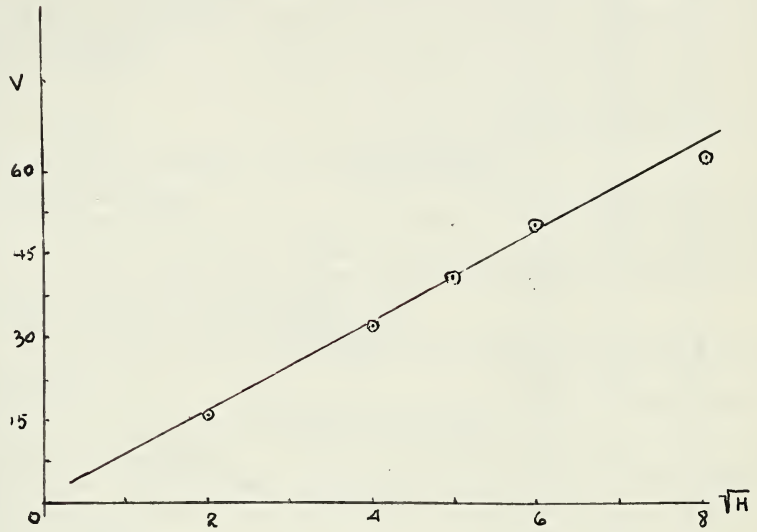
Question: 52 .

The connection between the speed V feet per sec. of a column of water produced by a pressure of head H feet, as determined experimentally, is shown in the following table.

H	4	16	25	36	64
V	15.7	32.7	39.3	49.1	63.3

Plot V against $\sqrt{H}$ and find the most probable relation expressing V in terms of H.

Correct Solution:



Marking Scheme:

1. Graphical work - 2 marks.
2. Coefficient of $\sqrt{H}$ (7.5 to 8.5) - 1 mark.
3. Constant term is 0 but allow $\pm$ 1 - 1 mark.
4. $V = 8\sqrt{H}$

TABLE CVI

Frequencies of Correct Answers, No Attempts & Errors
Question 53

	N	%
Correct Answers	35	11.7
No Attempts	19	6.3
No Graph	27	9.0
Confused attempt in calculating relationship	58	19.3
Constant term too large	41	13.7
Graph only-calculation of relationship not attempted	27	9.0
Assumed $V \propto H$	41	13.7
Points used in calculation too close together	3	1.0
Points chosen were not in the table	8	2.7
Answer in terms of x and y instead of $\sqrt{H}$ and V	21	7.0
Mechanical Errors e.g. addition subtraction	19	6.3
Plotted H against V	1	0.3

On the whole candidates showed little knowled of the process of finding the equation of a line having been given the points through which the line passed. Apparently, graph paper was not supplied in certain instances and as a result Candidates went to a lot of work to draw a graph, some even ruling a fine grid. 9% of the Candidates did not take time to draw a graph, possibly due to a shortage of time. Problem 7 on p. 146 of the text is similar.

CHAPTER VI

Comparison of Results With Previous Studies

For comparisons to be valid, they must be made between populations of equal ability and these populations under consideration must be subjected to the same tests.

From a study of previous investigations in Algebra 2 (Grade XII Algebra or Algebra 3) the following data reveal the groups or populations which were used in each study:

1. Albegra 3 Investigation of 1933 "A representative group of final examination papers written by Algebra 3 pupils in 1933". No mention is made of how this representative group was selected, nor in what range of marks this group fell.

2. An analysis of Errors made on the Departmental Examination in Algebra 3 of June 1935. The 600 cases used in this investigation were selected at random from approximately 1800, the total number of Algebra 3 papers written in the Province in June 1935.

3. A report on the Algebra 3 Paper of June 1938- The 329 papers were a sample consisting of every eighth paper in the % range 0 - 100.

4. A report on the Algebra 2 Papers of 1940, 1941 and 1942.

a. 1940-There were 297 papers in the sample lot for the 1940 Examination. No mention is made of the method of selection nor % range.

b. There were 309 papers in the sample lot for the 1941 examination. No mention of method of selection nor % range.

c. There were 322 papers in the sample lot for the 1942 examination. No mention of method of selection nor score range.

5. Student attainment in Algebra 2 Examination of 1944-One hundred of the cases used in the study were all those whose raw scores were 42, 43, 44, 45, 46 and 47 and as many students whose raw score was 48 as were required to bring the total number of papers up to 100. One hundred of the cases used in the study were all those whose raw scores were 65, 66, 67, 68 and 69 and as many students whose raw score was 70 as were required to bring the total number of papers up to 100. These raw scores corresponded to percentages 50 to 53.

6. An Investigation of the standards as set by the Grade XII Algebra 2 Examination of June 1948.

The 300 cases used in this study were all those whose raw scores were 68, 69, 70, 71, 72, 73 and as many students whose raw score was 74 as were required to bring the total number of papers up to 300. These raw scores corresponded to percentages 50 to 54.

Thus it is clear that the only valid comparison which can be made is between the 1948 study and the part of the 1944 study which deals with the 100 cases who just managed to get a B standing (50-64). These two groups then are similar in ability. However, it is evident that the two tests were not similar. Therefore, only those questions have been selected for comparison which are alike.

In the chapter dealing with conclusions (Chapter XI), the general conclusions of former studies will be compared with the results of this study.

The following table show the percentage of total marks which were allocated to different topics on the examination papers mentioned. It must be remembered that the 1933 study was concerned with the old Algebra 3 course which was based on Crawford's High School Algebra.

TABLE CVII

% of Examination Papers Assigned to Various Topic

Topic	1933	1935	1938	1940	1941	1942	1944	19
Exponents & Logarithms	-	-	-	-	-	1	1	
Ratio & Proportion	6	5	4	5	2	2	8	
Variation	8	11	4	7	9	9	4	
Functions & Graphs	6	15	4	18	18	26	18	2
Limits & Gradients	-	12	4	12	9	10	2	
Differentiation	-	11	16	13	14	9	16	1
Integration	-	11	13	8	12	9	10	
Arithmetic Progression	6	5	4	6	3	7	1	
Maxima-Minima	-	-	16	7	4	6	6	
Binomial Theorem	26	11	3	7	7	7	6	
Permutations Combinations	20	5	13	3	6	6	13	
Geometric Progression	20	-	4	-	4	2	7	
Annuities	8	7	6	8	6	6	4	
Empirical Formulae	-	7	9	6	6	-	4	
	100	100	100	100	100	100	100	10

Thus it is evident that the emphasis of instruction in the course has shifted from Binomial Theorem, Permutations and Combinations and Geometric Progressions in 1933 to Functions and graphs and Differentiation from 1935 on.

Comparison of Results - 1944 and 1948

In each instance, the two questions will be given as well as tables showing correct results and errors. The first 25 questions of the 1944 paper were of the short answer type i.e. a blank was left on the paper for the answer.

I a. Question 2-1948 The fourth proportional to 2, 3, and 4 is (a) 5, (b) 6, (c) 7, (d) 9, (e) 24-2marks

b. Question 2-1944 If 5,a,10,3 are in proportion, find a . Correct Answer: a = 1.5 - 2 marks.

TABLE CVIII a

Correct Scores

Question NO.	N[1]	%
2 - 1948	244	81.3
2 - 1944	91	91

1. In the 1944 study, 100 cases were considered, therefore N also represents the %.

The two questions, although not strictly the same are similar in that the candidate is required to find a missing proportion. From the data in Table Ia, pupils in 1944 were better able to do a problem of thi type.

TABLE CVIII b

Frequency of Errors and No Attempts

Year	Explanation	N	%
1944	a = 6	1	1
	a = $\sqrt{50}$	1	1
	a = 16 2/3	1	1
	Mechanical Errors	1	1
	Confused Attempts	5	5
	No Attempts	0	0
1948	5	55	18.3
	7	0	0.0
	9	1	0.4
	24	0	0.0
	No Attempt	0	0,0

The total % error in 1944 was 9 while in 1948 it was 18.7 i.e. there were twice a many errors in 1948 as in 1944. Of interest is the fact that these two problems were attempted by all pupils. It must be remembered that there were 100 cases in 1944 in this range as against 300 for 1948. Thus in 1944 N the frequency also represents the percent.

II a. Question 5 - 1948 The graph of the function $(x+2)(x-3)$ crosses the y axis at a. 3, b. 2, c. 6, d. -1, e. -6 - 2 marks

b. Question 4 - 1944 At what three points does the graph of the function $x^3 - x$ cross the x axis.

Correct Answer -1, o, +1.

The two questions, although not the same, test whether or not pupils have mastered the concept of a function crossing the coordinate axes i.e. when either x or y is zero.

TABLE CIX a

Comparison of Correct Scores

Question No.	N	%
5 - 1948	242	80.7
4 - 1944	77	77

TABLE CIX b

Comparison of Frequencies of Errors and No Attempts

Year	Explanation	N	%
1948	Gave answer as 3	43	14.4
	" " " 2	0	0.0
	" " " 6	7	2.3
	" " " -1	7	2.3
	No Attempts	1	0.3
1944	Gave two correct points	7	7.0
	" answer involving x, -x, x^3	2	2.0
	Gave factors $(x)(x-1)(x+1)$	1	1.0
	Other incorrect answers	6	6.0
	No Attempts	7	7.0

The results indicate possibly that the 1944 question is on the whole more difficult than the 1948 question. Then too, one must consider the elements involved in the two questions. The 1944 question involves recall while the 1948 question requires an element of recognition. Even with these factors involved, the results are not too much at variance as an examination of table IIa would indicate.

III a. Question 3-1948 Which ratio has the greatest value a. 11/12, b. 47/48, C. 5/6, d. 23/24, e. 35/36 - 2 marks.

b. Question 1-1944 What fraction with denominator 33 is equivalent to $\frac{1\frac{2}{3}}{11}$? Correct Answer: 5/33 - 2 marks.

TABLE CX a

Comparison of Correct Scores

Question No	N	%
3 - 1948	200	66.7
1 - 1944	75	75

The two question , although not the same, involve fraction work. As previously stated the 1948 question is found in the Grade VII text. Thus there is no reason whatsoever for Grade XII students even at B level to have difficulty with simple problems such as these. Results were better in 1944 than in 1948, but still one should expect 100% accuracy from students who have been in school for 12 years or more and whose next step is University.

TABLE CX b

Comparison of Frequencies of Errors and No Attempts.

Year	Explanation	N	%
1948	Gave answer as 11/12	3	1.0
	" " " 5/6	81	27.0
	" " " 35/36	13	4.3
	" " " 23/24	2	0.7
	No Attempts	1	0.3
1944	Language Difficulty- gave numerator only"5"	14	14
	Confused attempt e.g. 55/33	9	9
	No Attempts	2	2

IVa

Question 8-1948 If $f(x) = x^3 - x^2$, then the value of $f(-2)$ is equal to a) o, b. 12, c. 4, d. -12, e. -8 - 2 marks.

b. Question 8-1944 If $f(x) = \frac{x^2 + 1}{x}$, find the value of $f(-1)$.

Correct Answer: -2 - 2 marks

TABLE CXI a

Comparison of Correct Scores

Question No.	N	%
8 - 1948	286	95.3
8 - 1944	88	88

TABLE CXI b

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Gave answer as o	0	0.0
	" " " 12	1	0.3
	" " " 4	11	3.7
	" " " -8	2	0.7
	No Attempts	0	0.0
1944	Gave answer as +2	6	6
	" " " o	2	2
	" " " -1	2	2
	Other confused attempts	2	2
	No Attempts	0	0

The 1948 candidates were able to perform better on a problem dealing with functional notation than 1944 candidates. Of interest is the fact that all candidates attempted these two problems.

V a. Question 9-1948 If h = o, then the value of $\frac{2h+h}{h^2}$ is a. 2+h , b. undefined, c. 2, d. o, e.

2 marks

b. Question 10-1944 As n increases without limit, find the limit approached by $\frac{12}{n-10}$

Correct Answer - o - 2 marks

TABLE CXII a

Comparison of Correct Scores

Question No.	N	%
9 - 1948	58	19.3
10 - 1944	57	57

The two questions have been designed with a view of testing pupils knowledge of limits. In one case the value is asked for and in the other the limits. Thus there is some broad basis for comparison. However, it must be remembered that the answers were given in 1948 but not in 1944. Some pupils no doubt become conditioned to saying "undefined" wherever they meet the symbol o. On a broad basis then, 1944 students were better able to handle the concept of

limits as the results in table V a have indicated. A reason for the poor showing in 1948 may be due to the fact that pupils guessed.

TABLE CXII b

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Gave answer as 2+h	8	2.7
	" " " 2	120	40.0
	" " " o	96	32.0
	" " " ∞	16	5.3
	No Attempts	2	0.7
1944	Language difficulty: decreases without limit etc.	8	8.0
	Gave answer as 12	1	1.0
	Gave answer as $-\infty$	1	1.0
	Gave answer as $+\infty$	7	7.0
	Gave answer as $\frac{1}{\infty}$, $\frac{12}{\infty}$	3	3.0
	Gave answer as -1.2	5	5.0
	Gave answer as 1	3	3.0
	Gave answer as -1	2	2.0
	Other confused attempts	9	9.0
	No Attempts	7	7.0

As the analysis indicates, the answers given were many and varied. 1944 candidates were as much at sea over this concept as were 1948 candidates.

VI a. Question 11 - 1948 M varies directly as S. If M= 40 when s= 12, then when s= 18, M equals:

a. 6, b. 30, c. 60, d. 5.4, e 26 2/3 -- 2 marks

b. Question 3 - 1944 U and V are related variables. If doubling the variation- relation between U and V.

Correct Answer $U \propto \frac{1}{V}$ -- 2 marks

TABLE CXIII a

Comparison of Correct Scores

Question No.	N	%
11 - 1948	284	94.6
3 - 1944	. 71	71

The two questions were designed to test pupils knowledge of variation. The 1948 question deals with direct variation while the 1944 question deals with inverse variation. On the whole problem 3 - 1944 is much more difficult than problem 11 - 1948, as the results indicate. As before, only a broad basis of comparison is possible.

TABLE CXIIIb

Comparison of Erros and No Attempts

Year	Explanation	N	%
1948	Gave answer as 6	2	0.7
	" " " 30	2	0.7
	" " " 5.4	9	3.0
	" " " 26 2/3	2	0.7
	No Attempts	1	0.3
1944	Gave answer as $U^2 \propto \frac{1}{2V}$	3	3.0
	" " " $U \propto V$	2	2.0
	" " " $U \propto \frac{1}{4}V$ or $\frac{1}{2}kv$	12	12.0
	" " " $V = \frac{1}{U}$	2	2.0
	" " " $U \propto \frac{1}{2}v$	2	2.0
	" " " $U^2 \propto \frac{1}{V}$	13	13.0
	No Attempts	1	1.0

Of interest is the fact that only one candidate in each group did not attempt the problem for his group. Again, the fact that 1948 candidates had the answer before them might be given as a reason for so many correct responses.

VII a.

Question 12-1948 The gradient of the straight line which passes through the points (-4,6) and (3,-2) is:

a. -7/8, b. $\sqrt{17}$, c. $\sqrt{65}$, d. -8/7, e. 8/7 - 2 marks

b. Question 11-1944 Find the gradient of the straight line which has x and y intercepts, 3 and -2 respectively.

Correct Answer: 2/3 - 2 marks

TABLE CXIV a

Comparison of Correct Scores

Question No.	N	%
12 - 1948	244	81.3
11 - 1944	31	31

Again, there is possibly no basis for a comparison. Question 11 - 1944 is more difficult than question 12 - 1948. In 1948, the two points were given while in 1944, the points were given as intercepts, on the x and y axes. As results indicate, the 1948 candidates were much superior to the 1944 candidates in regard to finding the gradient of a straight line through 2 points.

TABLE CXIV b

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Gave answer as - 7/8	30	10.0
	" " " $\sqrt{17}$	0	0.0
	" " " $\sqrt{65}$	0	0.0
	" " " 8/7	17	5.7
	No Attempts	9	3.0
1944	Gave Answer as - 2/3	38	38.0
	" " " -3/2	6	6.0
	" " " 3/2	1	1.0
	Other incorrect answers	9	9.0
	No Attempts	15	15.0

Of interest in Table VII b. is the fact that 38% of the 1944 candidates had difficulty with signed numbers while 17% had the same difficulty in 1948.

VIII a. Question 13-1948 A simple function which has the following properties $f(2)=o$ and $f(-3) = o$ is:

a. x^2+5x+6, b. x^2-x+6, c. x^2+x-6 ,

d. x^2-3x+2, e. x^2+x+6 -- 2 marks.

b. Question 7 - 1944 The roots of the quadratic equation $x^2-6x+c = o$ are equal. Find the value of c.

Correct Answer : $c= 9$.

TABLE CXV a

Comparison of Correct Scores

Question No.	N	%
13 - 1948	270	90
7 - 1944	76	76

Again, the questions are not strictly similar. In 1948, the roots were given and it was an easy matter to substitute in the given equation to find the correct one. This is evidenced by the number of correct solutions. The 1944 question was relatively easy, and results should have been much better. Questions similar to the 1944 question appear in Grade X Algebra.

TABLE CXV b

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Gave answer as x^2+5x+6	2	0.7
	" " " x^2-x+6	11	3.7
	" " " x^2-3x+2	6	2.0
	" " " x^2+x+6	5	1.6
	No Attempts	6	2.0
1944	Gave Answer as $c=-3$	5	5.0
	" " " $c=+3$	3	3.0
	" " " $c=6x-x^2$	2	2.0
	" " " $c=6$	1	1.0
	" " " $c=0$	2	2.0
	Other confused attempts	6	6.0
	No Attempts	5	5.0

The number of candidates not attempting the two problems was about equal in the two years.

IX a. Question 14-1948 $\frac{d}{dx}$ (2x-4) (x+3) is :

a. 2, b. 4x+2, c. $2x^2$, d. 2x+2, e 4x. -- 2 marks

b. Question 12-1944 Given $y = 3x^5$. Find $\frac{dy}{dx}$.

Correct Answer: 15 x^4 -- 2 marks.

TABLE CXVI a

Comparison of Correct Scores

Question No.	N	%
14 - 1948	283	94.3
12 - 1944	97	97.0

Question 14 - 1948 is a little more difficult since it involves multiplying (2x-4) by (x+3) and as has already been pointed out numerous algebraic multiplication mistakes were made by this group. Question 12 - 1944 is a great deal easier, and as a result the 1944 group holds a slight advantage over the 1948 group.

TABLE CXVI b

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Gave answer as 2	10	3.4
	" " " $2x^2$	0	0.0
	" " " $2x+2$	5	1.7
	" " " $4x$	1	0.3
	No Attempts	1	0.3
1944	Gave Answer as $15x^2$	2	2.0
	Other Errors	5	5.0
	No Attempts	0	0.0

X a. Question 19-1948 $\int_4^9 \sqrt{x}\, dx$ is equal to :

a. 3 1/3, b. 1, c. 15 1/3, d. 19, e. 12 2/3 - 2 marks

b. Question 16-1944 Evaluate $\int_{-1}^{+1} 3x^2\, dx$

Correct Answer 2 - 2 marks

TABLE CXVII a

Comparison of Correct Scores

Question No.	N	%
19 - 1948	145	48.3
16 - 1944	72	72

These two questions have been designed to test the candidates knowledge of definite integrals. There is no doubt but that the 1948 question is on the whole a little more difficult. However, the 1944 group holds a decided advantage over the 1948 group.

TABLE CXVII b

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Gave answer as 3 1/3	32	10.7
	" " " 1	54	18.0
	" " " 15 1/3	21	7.0
	" " " 19	20	6.7
	No Attempts	28	9.3
1944	Gave answer as o	22	22.0
	" " " 12	2	2.0
	Other incorrect answers	4	4.0
	No Attempt	0	0.0

Of significance here is the comparison of No Attempts for the two questions. Everyone in this group attempted the question in 1944 while 9.3% did not attempt it in 1948.

XI a Question 20-1948 The tenth and eleventh term of Arithmetic Progression are 53 and 56 respectively. The fifth term is:
a. 26, b. 3, c. 35, d. 38, e. 41 -- 2 marks

b. Question 17-1944 The first and second terms of an A.P. are a and b . What is the third term?

Correct Answer : 2b-a: 2 marks

TABLE CXVIII a

Comparison of Correct Scores

Question No	N	%
20 - 1948	241	80.3
17 - 1944	57	57

The 1944 question is on the whole more difficult than the 1948 question. Pupils of this group are well able to handle numerical cases as is illustrat by the good showing made by the 1948 candidates. It is always more difficult to work in terms of algebraic quantities for pupils of this category. On a broad basis of comparison, then, 1948 candidates were better able to handle this type of problem.

TABLE CXVIIIb

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Gave answer as 26	15	5.0
	Gave answer as 3	3	1.0
	" " " 35	18	6.0
	" " " 41	12	4.0
	No Attempts	11	3.7
1944	Gave answer as a+2b/a	3	3.0
	" " " a+b	2	2.0
	Incomplete&other answers	18	18.0
	Gave answer as 3a-2b	7	7.0
	" " " a+ b/a	1	1.0
	" " " a+2d	3	3.0
	" " " a+2b	5	5.0
	" " " c	2	2.0
	No Attempts	2	2.0

XII a. Question 24-1948 How many different 3 member executives consisting of president, vice-president and secretary-treasurer can be formed from a group of 10 persons?

a. 3, b. 120, c. 10^3, d. 72, e. 720 - 2 marks

B. Question 21-1944 Evaluate ${}^6P_6$

Correct Answer: 720 - 2 marks

TABLE CXIX a

Comparison of Correct Answers

Question No.	N	%
24- 1948	109	36.3
21- 1944	87	87.0

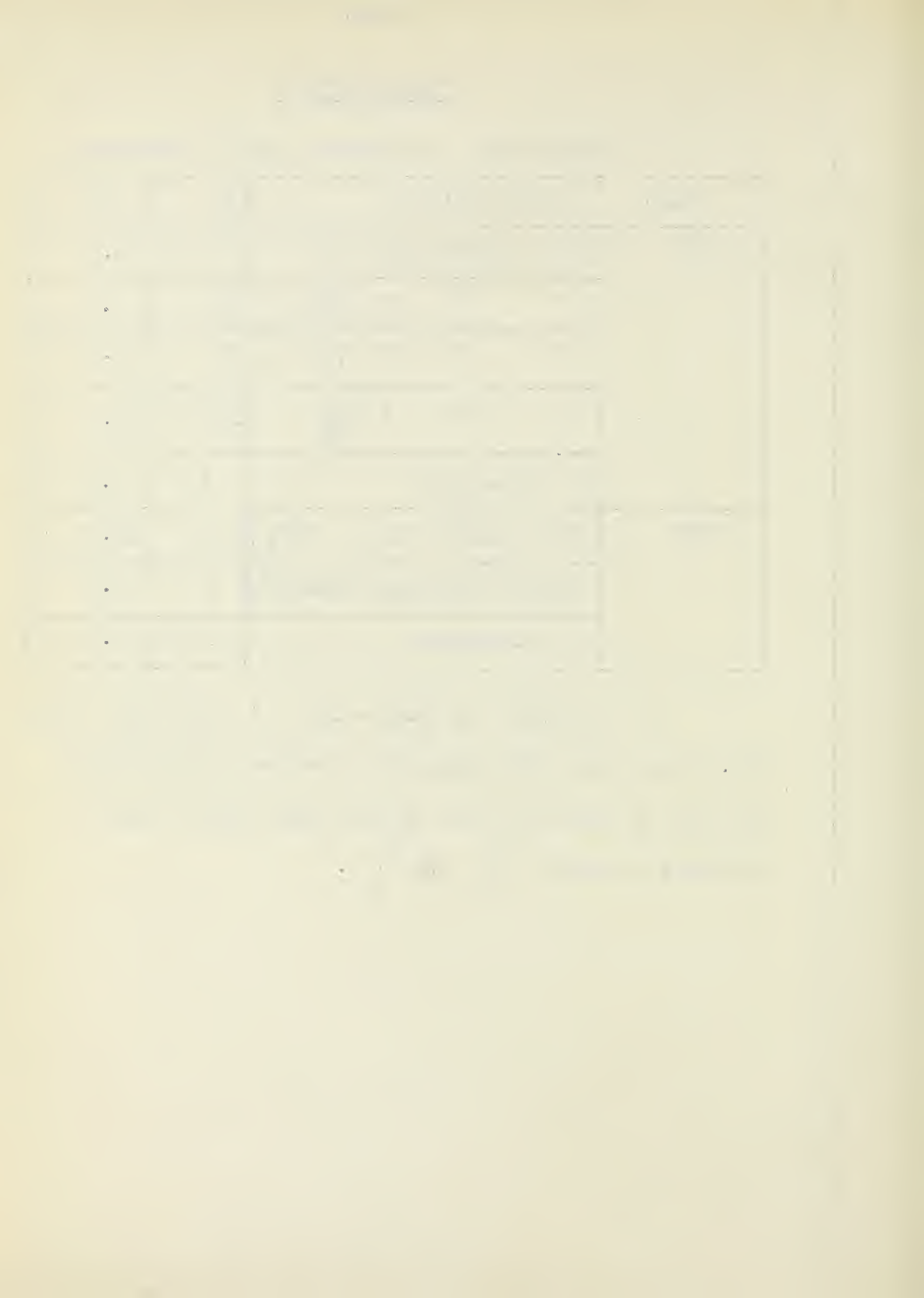
There is no doubt but that question 24 - 1948 is the more difficult, and it is only on the broad basis of evaluation of ${}^6P_6$ in the 1948 paper and ${}^{10}P_3$ in the 1944 paper that any valid comparison can be made.

TABLE CXIX b

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Gave answer as 3	0	0.0
	" " " 10^3	5	1.7
	" " " 72	2	0.7
	" " " 120 $^{10}C_3$	181	60.3
	No Attempts	3	1.0
1944	Gave answer as 1 ($^6C_6$)	10	10.0
	Other incorrect answers	2	2.0
	No Attempts	1	1.0

Of interest in Table XII b is the fact that 60.3 % of the 1948 students treated the problem as one in combinations while only 10% of 1944 students treated $^6P_6$ as $^6C_6$.



XIII a. Question 25 - 1948 If the first term of a Geometric Series is 5, the last term is 320 and the ratio is 2, then the number of terms is:

a. 5, b. 6, c. 7, d. 8, e. 9. - 2 marks

b. Question 18-1944 The second term of a G. P. is 10 and its common ratio is r. The first term is?

Correct Answer 10/r - 2 marks

TABLE CXX a

Comparison of Correct Scores

Question No.	N	%
25 - 1948	144	48.0
18 - 1944	63	63.0

The two problems are not similar. The 1948 question requires the number of terms while the 1944 questions requires the candidate to find the first term. Both questions have been designed however, to test pupils knowledge of Geometric Series and Progressions.

TABLE CXX b

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Gave answer as 5	23	7.7
	" " " 6	52	17.3
	" " " 8	23	7.7
	" " " 9	17	5.6
	No Attempts	41	13.7
1944	Gave answer as 10 r	4	4.0
	" " " 10 -r	2	2.0
	" " " 1	4	4.0
	" " " r/10	2	2.0
	" " " 10/a	3	3.0
	Others	17	17.0
	No Attempts	5	5.0

XIV a. Question 26-1948 How many committees of three can be formed from a group of 12 men?

a. 1320, b. 660, c. 440, d. 220, e. 110 -- 2 marks

b. Question 22-1944 In how many ways can three things be chosen from seven things?

Correct Answer: 35 -- 2 marks

TABLE CXXI

Comparison of Correct Scores

Question No.	N	%
26 - 1948	186	62.0
22 - 1944	59	59.0

The two questions are very similar. From an examination of results it is clear that the 1948 candidates show they were able to do as well as the 1944 group generally speaking.

TABLE CXXI b

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Gave answer as 1320($^{12}P_3$)	93	31.0
	" " " 660	7	2.3
	" " " 440	3	1.0
	" " " 110	3	1.0
	No Attempts	8	2.7
1944	Gave Answer as 210($^{7}P_3$)	30	30.0
	" " " 840	1	1.0
	Mech. Errors in evaluation	10	10.0
	No Attempts	0	0.0

Roughly the same % of candidates treated their respective problem as one in permutations rather than combinations. The 1944 question was attempted by everyone, but 2.7% of the candidates in 1948 did not attempt their problem.

XV a. Question 28-1948 - The coefficient of a^3 in $(a+1)^7$ is:

a. 3, b. 21, c. 30, d. 35, e. 42 -- 2 marks.

b. Question 23-1944 - What is the first term in the expansion of $(2x^3+y)^5$?

Correct Answer $32\ x^{15}$ - 2 marks

TABLE CXXII a

Comparison of Correct Scores

Question No	N	%
28-1948	221	73.7
23-1944	35	35.0

Of the two questions, the 1944 question was easier than the 1948 question. The 1948 question required the application of the formula for the Binomial Expansion, and more work was entailed and as a result there was an opportunity for more errors. The 1948 group showed a decided superiority over the 1944 group.

TABLE CXXIIb

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Gave answer as 3	16	5.3
	" " " 21	38	12.7
	" " " 30	7	2.3
	" " " 42	5	1.7
	No Attempts	13	4.3
1944	Gave answer as $(2x^3)^5$	8	8.0
	" " " $32x^8$	10	10.0
	" " " $2x^{15}$	6	6.0
	" " " $2x^8$	6	6.0
	" " " $2x^3$	6	6.0
	Other incorrect answers	25	25.0
	No Attempt	3	3.0

XVI a. Question 29-1948 If $Y = x^2$ what is the error in the approximation $\frac{\Delta y}{\Delta x} \simeq \frac{dy}{dx}$:

a. Δx, b. Δx^2 , c. $x\Delta x$, d. $2x\Delta x$, e.x - 2 marks.

b. Question 14-1944 Given $S = \frac{t^3}{3} + 1$ and assuming that $\frac{ds}{dt}\Delta t$ is an approximation for Δs, find the approximate change in S when t increases from 2.9 to 3.1.

Correct Answer $\Delta s \simeq 1.682$ - 2 marks

TABLE CXXIIIa

Comparison of Correct Scores

Question No.	N	%
29 - 1948	110	36.7
14 - 1944	24	24.0

The two questions are different in that the 1948 question is algebraic and taken directly from the text book while the 1944 question is numerical. Then too the answers were given for the 1948 question. The results show that the 1948 group was superior to the 1944 group in the topic dealing with approximations.

TABLE CXXIIIb

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Gave answer as Δx^2	81	27.0
	" " " $x\Delta x$	10	3.3
	" " " $2x\Delta x$	82	27.3
	" " " X	4	1.4
	No Attempts	13	4.3
1944	Went as far as $0.2t^2$ only	8	8.0
	Went as far as $t^2.\Delta t$ only	3	3.0
	Evaluated $.2t^2$ at 3.1 (not 2.9)	6	6.0
	Evaluated $.2t^2$ at 3.1 and 2.9	9	9.0
	Error in Evaluation	26	26.0
	Confused Attempts	15	15.0
	No Attempts	9	9.0

XVII a. Question 31-1948 The sum to infinity of a Geometricl Series is 24 and the first term is 12. The correct series is:

A. 12+4+1.33 +.... B. 12+2+o.33+....

C. 12+6+3+.... D. 12+2.4+0.48+....

E. 12+1.2+o.12+..... 2 marks.

b. Question 40 a-1944 Find the sum to infinity of the following series 1-1/2 +1/4- 1/8 + ...

Corect Answer S = 2/3 -- 3 marks

TABLE CXXIVa

Comparison of Correct Scores

Question No.	N	%
31 - 1948	215	71.7
40 a - 1944	49	49.0

The two questions are similar in that they treat with the sum of a G. P. to infinity. In the 1948 problem, the data are given and it is required to select the correct series while in the 1944 problem the data had to be abstracted from the series. On the whole the 1944 question with its common ratio equal to $-\frac{1}{2}$ provided an extra opportunity for errors while the 1948 question did not have a negative common ratio. Again the 1948 students showed superiority.

TABLE CXXIV b

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Gave answer as a.	16	5.3
	" " " b.	8	2.7
	" " " d.	17	5.7
	" " " e.	17	5.7
	No Attempts	27	9.0
1944	Gave answer as 2	11	11.0
	Used incorrect formula	10	10.0
	Mechanical Error	2	2.0
	Gave answer as 1	2	2.0
	" " " -1	1	1.0
	" " " $\frac{1}{2}$	2	2.0
	Confused with A.P.	2	2.0
	Gave answer as -2	2	2.0
	Incomplete	2	2.0
	Confused Attempt	7	7.0
	No Attempt	12	12.0

XVIII a. Question 33 - 1948 The altitude (a) of a triangle is one-half the length of the base. Express the area of the triangle as a function of (a).

a. $\sqrt{2a^2}$, b. $\sqrt{5/4\ a^2}$, c. a^2 , d. $a^2/4$, e. 2a - 2 marks.

b. Question 5 - 1944 Express the area (A) of a square as a function of its perimeter (p).

Correct Solution $A = \left(\frac{p}{4}\right)^2 = \frac{p^2}{16}$ - 2 marks.

TABLE CXXV a

Comparison of Correct Scores

Question No.	N	%
32 - 1948	198	66.0
5 - 1944	39	39.0

The two questions have been chosen for comparison because they both ask for the expression of area as a function of a given side. The 1944 question is possibly a bit more subtle because the side has to be found from the perimeter. The 1948 question on the other hand involves a reading problem for the candidate even though the side is given directly. Again the answer to the 1948 question was in front of the candidate and this may account for the relatively large number of correct responses as compared with the 1944 group. From an examination of the responses given for the 1944 question, it is evident that if the correct answer had been given the candidate he could have eliminated a great many of the answers which he got.

TABLE CXXVb

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Gave Answer as $\sqrt{2a^2}$	7	2.3
	" " " $\sqrt{5/4a^2}$	2	0.7
	" " " $a^2/4$	72	24.0
	" " " 2a	10	3.3
	No Attempts	11	3.7
1944	Gave answer as $A=p^2/4$	7	7.0
	" " " $A=p/2$	2	2.0
	" " " $A=p^2/2$	2	2.0
	" " " $A=p/4$	2.	2.0
	" " " $A=2p$	1	1.0
	Incomplete	10	10.0
	Confused Attempts	28	28.0
	No Attempts	8	8.0

XIX a. Question 36-1948 The safe load "W" (tons) for a beam varies jointly as the thickness "t" (inches) and the square of the depth "d" (inches) and inversely as the length "ℓ" (feet) between the supports. If the safe load for a beam 6 inches thich, 10 inches deep and 12 feet long is 5 tons, express W in terms of t, d and ℓ -

Correct Solution $W = \frac{td^2}{10\ell}$ - 4 marks.

b. Question 28-1944 The electrical resistance of a wire varies directly as its length and inversely as the square of its diameter. If the resistance of an 8 foot wire having a diameter of 1/10 of an inch is 2 ohms, find the resistance of a 10 foot wire made of the same material and having a diameter of 1/16 of an inch.

Correct Solution 6.4 ohms - 6 marks.

TABLE CXXVIa

Comparisons of Correct Scores.

Scores	1948[1]		1944[2]	
	N	%	N	%
0	75	25.0	8	8.0
1	69	23.0	1	1.0
2	10	3.3	15	15.0
3	16	5.4	4	4.0
4	130	43.3	8	8.0
5			6	6.0
6			58	58.0

1. Total Marks for problem - 4

2. Total Marks for problem - 6

These two questions dealing with joint variation are similar up to the point where the candidate has to find the variation constant. The 1944 question involves the application of the formula which the candidate has derived while the 1948 question does not require this. On the whole candidates in 1944 were superior to the 1948 group as is evidenced from Table XIX a.

TABLE CXXVIb

Frequencies of Errors and No Attempts

Year	Explanation	N	%
1948	Solved for k in terms of W,t,d, l, and did not substitute numerical value.	11	3.7
	Misread problem e.g. square root for square	15	5.0
	$W \propto 1/12\ell$ instead of $W \propto 1/\ell$ - changed units	63	21.0
	Mechanical Errors	26	8.7
	Correct Solution-Error on markers part in not awarding total marks.	2	0.7
	Miscellaneous -Translation Errors, did not reduce result etc.	13	4.3
	Inability to solve for k i.e. did not include the proportionality constant in the relation	33	11.0
	No Attempt	7	2.3
1944	Error in finding value of k	23	23.0
	$r \propto \frac{l}{\sqrt{d}}$ i.e. misread problem	1	1.0
	Other incorrect answers	10	10.0
	No Attempt	1	1.0

The number who did not attempt the problem was greater in 1948 than in 1944. Mechanical Errors were greater in 1944 than in 1948, while 5% misread the problem in 1948 as compared to 1% for 1944.

XX a. Question 37-1948 Using the axes below sketch the graph of the function $\frac{x+1}{(x-1)(x+2)}$

(For a correct solution see Chapter VI) - 6 marks.

b. Question 29-1948 Sketch the graph of the function $\frac{3x-2}{2x+3}$ using the set of axes provided below. - 6 marks.

TABLE CXXVIIa

Comparison of Correct Scores

Score	1944		1948	
	N	%	N	%
0	14	14.0	2.6	8.7
1	14	14.0	15	5.0
2	15	15.0	30	10.0
3	11	11.0	56	18.6
4	15	15.0	35	11.6
5	9	9.0	40	10.7
6	22	22.0	98	32.7

The two questions are very similar. Results indicate a limited knowledge of graph sketching by both groups. On the whole these problems were too difficult for both groups. From an examination of the data, 1948 candidates were probably superior to 1944 candidates in graph sketching.

TABLE CXXVIIb

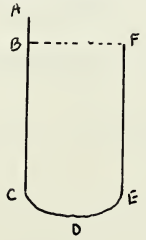
Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Placed extra branches of the function in the (-x,y) and (x,-y) quadrants	7	2.3
	Analyzed the function as a parabola	38	12.7
	Inverted middle section of function or could not sketch it	56	18.6
	Left out side branches	24	8.0
	Confused attempt	38	12.7
	Put in horizontal asymptote at y=1 and could not sketch central section	6	2.0
	Sketch not accurate	36	12.0
	No Attempts	3	1.0
1944	Horizontal asymptote missing or incorrect	64	64.0
	Vertical Asymptote missing or incorrect	41	41.0
	x and y intercepts incorrect or not indicated	9	9.0
	No Attempt	2	2.0

XXI a. Question 39-1948 - An open cardboard box with square ends is fitted with an overlapping lid which covers the open top, the whole of two square ends and one other side. The total area of the cardboard in box and cover is eight square feet. What is the length of the square end so that the volume of the box will be a maximum?

Correct Answer $\sqrt{\frac{2}{3}}$ or 0.82 or 0.816 feet-6mark

b. Question 38-1944 A long rectangular strip of sheet iron 24 inches wide is made into an eaves-trough with cross section as shown in the diagram. The straight sides AC and EF are unequal, BC=EF and AB is one half the radius of the semi circular base CDE. Find to the nearest tenth of an inch the width of the cross section for which the trough will have greatest water holding capacity. Show at the proper place in your solution, that you have a "greatest" and not a "least.".



Correct Answer : 11.6 inches - 7 marks.

TABLE CXXVIIIa

Comparison of Correct Scores

Score	1948 [1.]		1944 [2.]	
	N	%	N	%
0	208	69.3	99	99.0
1	31	10.4	0	0.0
2	36	12.0	1	1.0
3	15	5.0	0	0.0
4	7	2.3	0	0.0
5	1	0.3	0	0.0
6	2	0.7	0	0.0
7			0	0.0

1. Total Score 6

2. " " 7

From the table it is evident that the 1944 question is the more difficult of the two and there is no valid basis for a comparison. Both groups performed very poorly on this question. However, the 1948 group were able to achieve more partial scores than the 1944 group.

TABLE CXXVIIIb

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Found volume could go no farther	4	1.3
	No proof for maximum value	7	2.3
	Used too many unknowns	7	2.3
	Preliminaries only, drew figure etc.	62	20.7
	Wrong analysis(see Chapt.VI)	85	28.3
	Solved correctly for length in terms of area - could go no further	52	17.3
	Translation Error	5	1.7
	Mechanical Errors	26	8.7
	Should have had more marks	19	6.4
	Miscellaneous	3	1.0
	No Attempt	28	9.3
1944	Mechanical Errors	4	4.0
	Incomplete	14	14.0
	Confused Attempt	21	21.0
	No Attempt	61	61.0

XXII a. Question 41-1948 Find the value of $\frac{8132h^{2.5}}{n^2}$ when h= 4.65 and n = 325, correct to two decimals.

Correct Answer - 3.59 - 4 marks.

b. Question 19-1944 Using pages 30-31 of Knott's tables, find the cube root of 200

Correct Answer 5.8480 - 2 marks.

TABLE CXXIXa

Comparison of Correct Scores

Scores	1948 [1]		1944 [2]	
	N	%	N	%
0	151	50.3	45	45.0
1	10	3.3	0	0.0
2	29	9.7	55	55.0
3	48	16.0		
4	62	20.7		

1. Total Marks 4

2. Total Marks 2

The two questions are not similar and a basis for comparison rests solely on the ability of candidates to use tables. The value of 200 in the 1944 question can be found directly from the tables. Eve so, only 55% were able to do so. The 1948 questions required logarithmic calculation.

TABLE CXXIXb

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Substitution made-no other work attempted	31	10.3
	Error in indices	32	10.7
	Translation Errors	26	8.7
	Mechanical Errors	24	8.0
	Significant figures	30	10.0
	Did not use logs to evaluate	25	8.3
	Multiplied logs	6	2.0
	Error in looking up logs	41	13.7
	Error in loo ing up anti-logs	9	3.0
	No Attempt	14	4.7
1944	5.85 - not accurate enough (Sig. figs.)	8	8.0
	27.144 (used 10 $\sqrt[3]{20}$)	7	7.0
	2.7114 (used $\sqrt[3]{20}$)	1	1.0
	9.2832 ($2\sqrt[3]{200}$)	4	4.0
	12.559 ($10\sqrt[3]{2}$)	1	1.0
	Cube instead of cube root (8,000,000)	2	2.0
	14.142 ($10\sqrt{2}$)	3	3.0
	Other incorrect answers	9	9.0
	No Attempt	9	9.0

XXIII a. Question 44-1948 - The figure below represents a solid formed by rotating part of the graph of $y = x^2 + 1$ about the x axis. (See Chapter VI for figure) Find the volume to the nearest whole number.

Correct Answer: 37 Cubic Units - 6 marks.

b. Question 39-1944 As shown in the accompanying diagram, a portion AB of the graph of the function $x^2 + 2$ has been rotated about the x axis. Find the volume of the solid so generated, giving your answer to the nearest whole number.

Correct Answer: 321 cubic units - 6 marks.

TABLE CXXXa

Comparison of Correct Scores.

Scores	1948		1944	
	N	%	N	%
0	152	50.7	55	55.0
1	25	8.3	2	2.0
2	29	9.7	8	8.0
3	28	9.3	19	19.0
4	29	9.7	9	9.0
5	17	5.7	4	4.0
6	20	6.6	3	3.0

The two questions are quite similar. Examination of Table XXIIIa shows that on a broad basis of

comparison, the 1948 candidates were able to perform better than the 1944 candidates in integrating to find a volume.

TABLE CXXXb

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Used incorrect formula to find volume e.g. $\pi \int^{2}(x^2+1)dx$	54	18.0
	Mechanical Errors	58	19.4
	Did not multiply result by	22	7.3
	Translation Error	13	4.3
	Did not Integrate	8	2.7
	Error in limits or could not find limits	61	20.3
	Differentiated	21	7.0
	Did not bring result to nearest whole number	7	2.3
	Did not evaluate at upper or lower limit	4	1.3
	Integrated before squaring	3	1.0
	Should have had more marks	3	1.0
	Incomplete-right as far as the pupil went.	5	1.7
	No Attempt	21	7.0
1944	Incorrect integral	35	35.0
	Incorrect evaluation	34	34.0
	Omitted π	21	21.0
	$\int (x^2+2)dx$	14	14.0
	Error in Limits	9	9.0
	Incomplete	2	2.0
	Others	9	9.0
	No Attempt	14	14.0

XXIV a. Question 45-1948 Solve the Equation

$5 + \sqrt{x-2} = 2x$.

Correct Answer : $x = 3$ - 5 marks

b. Question 9-1944 Solve the equation

$2x^2 - 3x - 4 = 0$, giving the roots correct to the nearest second decimal place.

Correct Answer: $x = 2.35$, 0.85 - 2 marks.

TABLE CXXXIa

Comparison of Correct Scores

Scores	1948 1.		1944 2.	
	N	%	N	%
0	228	76.0	67	67.0
1	17	5.7	11	11.0
2	18	6.0	22	22.0
3	4	1.3		
4	28	9.3		
5	5	1.7		

1. Total Score 5,

2. Total Score 2.

The two questions are dissimilar and there is no doubt but that the 1948 question is more difficult. However, the showing of the 1944 candidates is not very good. Surprising enough is the fact that students leave Grade X, after working dozens of examples in the solution of equations, only to come to Grade XII and regard the solution of equations as something which they had never seen. Results are very poor for both groups.

TABLE CXXXIb

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Error in evaluation of $\sqrt{b^2-4ac}$	5	1.6
	Did not transpose-squared equation as given	121	40.3
	9/4 not listed as extraneous	32	10.7
	Attempted to use the formula directly	15	5.0
	Mechanical Errors in Arithmetic and Algebra	41	13.7
	Miscellaneous-Integrated, differentiated, used Binomial Theorem	16	5.3
	Incomeplete	21	7.0
	Translation Error	8	2.7
	Guessed at one of the roots	3	1.0
	Transposition correct - could not factor	6	2.0
	Expressed x as a function x	4	1.3
	Squared (x-2)	3	1.0
	Squared $(5+\sqrt{x-2})$ then could not factor	8	2.7
	No Attempts	12	4.0
1944	Formula incorrectly used	14	14.0
	Mech. Error in evaluating	17	17.0
	Other Errors in evaluation	14	14.0
	Tried to factor	7	7.0
	Tried to complete square	6	6.0
	Reversed signs	6	6.0
	Gave one correct value only	11	11.0
	Other confuesed attempts	2	2.0
	No Attempts	8	8.0

XXV a. Question 46-1948 The first term of a Geometric Progression is 1, the last term is 512 and the common ratio is 2. Find the number of terms.

Correct Answer:-n = 10 - 3 marks

b. Question 30-1948 If $y = 3\ (1.05)^{x}$ show that when x = 2,4,6,8 the corresponding values of y are in G.P. and fine the common ratio.

Correct Answer: $r = (1.05)^{2}$ - 4 marks

TABLE CXXXIIa

Comparison of Correct Scores

Scores	1948		1944	
	N	%	N	%
0	143	47.7	60	60.0
1	54	18.0	12	12.0
2	39	13.0	8	8.0
3	64	21.3	7	7.0
4			13	13.0

Strictly speaking, these two questions are not similar. They have been chosen for their similarity in that they deal with Geometric Progressions. The 1944 question is a growth function. As results indicate, these questions were poorly done.

TABLE CXXXIIb

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Attempted to use formula for the sum of a Geometric Series	36	12.0
	Wrote down terms-wrong results	5	1.7
	Mechanical Errors	12	4.0
	Could not solve $2^{n-1} = 512$	32	10.7
	Wrote down terms-correct result	38	12.7
	Treated the question as one in Arithmetic Progression	18	6.0
	Correct start - incomplete	12	4.0
	Should have had more marks	8	2.7
	Confused Attempt	27	9.0
	$2^n = 512$	7	2.3
	Translation Error	7	2.3
	Attempted to use Binomial Expansion	1	0.3
	No Attempt	33	11.0
1944	Evaluated each y and found one ratio but was inaccurate	19	19.0
	Evaluated y incorrectly	14	14.0
	Mechanical Error	9	9.0
	Found r incorrectly (subtacted)	6	6.0
	Correct answer - method not clear	10	10.0
	Incomplete	9	9.0
	No Attempts	15	15.0

XXVI a. Question 48-1948 An annuity of $100.00, being now (first payment due one year from now) has 15 years to run. It is left uncollected by the payee and accumulates at the rate of 2% compounded annually. Find the value of the annuity at maturity.

Correct Answer $1729.35 - 4 marks

b. Question 41-1944 On Dec. 1, 1935 a man invested $500.00 and continued to invest this same amount on each succeeding Dec. 1. If these deposits earn interest at the rate of 4% compounded annually, find the total amount to his credit immediately after he has made his deposit on Dec. 1, 1944.

Correct Answer $6,0003 - 6 marks.

TABLE CXXXIIIa

Comparison of Correct Scores

Scores	1948 [1] N	%	1944 [2] N	%
0	160	43.3	78	78.0
1	11	3.7	5	5.0
2	10	3.3	1	1.0
3	21	7.0	5	5.0
4	98	32.7	2	2.0
5			3	3.0
6			6	6.0

1. Total marks 4

2. Total marks 6

TABLE CXXXIIIb

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Misread Problem-worked as a problem in Present Worth	45	15.0
	Expressed answer to the nearest dollar	8	2.7
	Summed to 14 or 16 terms	15	5.0
	Error in setting down series	15	5.0
	Read problem as being in compound interest	42	14.0
	Mechanical Errors	20	6.6
	Confused Attempt	8	2.7
	Error in applying formula for sum of a Geometric Series	10	3.3
	Translation Error	3	1.0
	Incomplete	11	3.6
	Should have had more marks	2	0.7
	No Attempt	23	7.7
1944	Summed to 9 terms only	31	31.0
	Worked as a Problem in Present Worth	15	15.0
	$500\ (1.04)^9$ - Compound Int.	17	17.0
	Error in Evaluation	9	9.0
	First term 500 (1.04)	3	3.0
	Confused Attempt	17	17.0
	Summed two terms only	5	5.0
	No Attempt	10	10.0

These two questions are very similar. However, the directions as given in the 1948 question: (first payment due one year from now) may account for the relatively good standing of the 1948 candidates as compared to the 1944 candidates. However, if pupils are taught to draw a time line for questions of this sort then there would be very little difficulty in finding the correct number of years that an annuity has to run. On the whole, the showing of 1944 pupils is inferior to that of the 1948 candidates. It is interesting to note that in each year 15% of the candidates attempted their respective problem as one in Present Worth while the number of confused attempts was higher in 1944 than in 1948. Mechanical errors accounted for 26% of the candidates losing marks in 1948 while only 9% lost marks on this account 1944.

XXVII a. Question 49-1948 Eight flags, three of which are white, three red and two green are all displayed in a column. How many different signals can be shown?

Correct Answer: 560 signals - 3 marks

$$\left(\frac{8!}{3!\ 3!\ 2!} \right)$$

b. Question 42-1944 Ten members of one club and twelve members of another meet to choose a committee of three. If both clubs must be represented the committee, how many different committees are possible?

Correct Answer: 1200 - 6 marks.

$$\left({}^{10}C_1 \times {}^{12}C_2 + {}^{12}C_1 \times {}^{10}C_2 \right)$$

The two questions are not similar. They have been shosen for comparison on the basis that they deal with the section of the text dealing with Permutations and Combinations.

TABLE CXXXIVa

Comparison of Correct Scores

Scores	1948 1.		1944 2.	
	N	%	N	%
0	297	99.0	69	69.0
1	0	0.0	0	0.0
2	0	0.0	2	2.0
3	3	1.0	1	1.0
4			3	3.0
5			2	2.0
6			23	23.0

1. Total Marks 3

2. Total Marks 6

As results indicate, pupils of both years made an extremely poor showing on the question. The 1948 question was more involved than the 1944 question. Thereis some doubt as to the advisability of teaching this section of the text. Invariably candidates have difficulty with understanding the concepts involved. Possibly this time could be spent to better advantage on other portions of the course. However, there is no denying the value of this section of the course to students who will want to go on to further study.

TABLE CXXXIVb

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	$^{5}C_{3} \times {}^{5}C_{3} \times {}^{6}C_{2}$	6	2.0
	$^{8}P_{2} + {}^{5}P_{3} + {}^{2}P_{2}$	7	2.3
	Drew flags, doodled etc.	22	7.3
	$^{6}C_{3} + {}^{6}C_{5} + {}^{7}C_{2}$	22	0.7
	$^{8}C_{3} + {}^{8}C_{3} + {}^{8}C_{2}$	56	18.6
	$^{8}C_{3}$	18	6.0
	$^{3}P_{3} + {}^{3}P_{3} + {}^{3}P_{2}$	11	3.7
	$^{8}P_{3} + {}^{8}P_{3} + {}^{8}P_{2}$	37	12.3
	8!	69	23.0
	$^{8}C_{3} \times {}^{5}C_{3}$	2	0.7
	$^{8}C_{3} + {}^{8}C_{2} + {}^{8}C_{1}$	5	1.7
	$^{8}C_{3} + {}^{5}C_{3} + {}^{5}C_{2}$	12	4.0
	8+7+6+5+4+3+2+1	2	0.7
	$^{3}C_{1} \times {}^{3}C_{1} \times {}^{2}C_{1}$	11	3.7
	$^{6}P_{3} \times {}^{6}P_{2}$	5	1.7
	3! X 3! X 2!	7	2.3
	$^{8}P_{5} + {}^{8}P_{5} + {}^{8}P_{6}$	1	0.3
	3^{8}	2	0.7
	$8^{3} + 7^{3} + 6^{2}$	2	0.7
	$^{8}C_{2} \times {}^{8}C_{3} \times {}^{8}C_{3}$	10	3.3
	No Attempt	10	3.3

cont'd.

TABLE CXXXIVb
concluded

Comparison of Errors and No Attempts

Year	Explanation	N	%
1944	O.K. as far as $^{10}C_1 \times {}^{12}C_2 + {}^{12}C_1 \times {}^{10}C_2$	8	8.0
	$^{10}C_1 + {}^{12}C_1 + {}^{10}C_2 + {}^{12}C_2$	9	9.0
	Mechanical Error	6	6.0
	Used Permutations instead of Combinations	6	6.0
	$^{10}C_3 \times {}^{12}C_3$	7	7.0
	$^{10}C_3 + C_3$	2	2.0
	$^{10}C_1 \times {}^{12}C_1 \times {}^{10}C_2 \times {}^{12}C_2$	6	6.0
	$^{22}C_3$	5	5.0
	2400	5	5.0
	Incomplete	4	4.0
	Confused Attempts	17	17.0
	No Attempt	7	7.0

XXVIII a. Question 50-1948 Show by using the binomial Theorem that the cube root of 28 is approximately 3.037.

Correct Answer: 3.037 - 4 marks.

b. Question 24-1944 Assuming that under certain conditions $(1+x)^n \simeq 1+nx$, find an approximate value for $\sqrt[3]{1.06}$

Correct answer; 1.02 - 2 marks

TABLE CXXXVa

Comparison of Correct Scores

Scores	1948 1.		1944 2.	
	N	%	N	%
0	215	71.7	22	22.0
1	13	4.3	0	0.0
2	13	4.3	78	78.0
3	9	3.0		
4	50	16.7		

1. Total marks 4

2. Total marks 2

The two questions are similar in that the Binomial Approximation $(1+x)^n \simeq 1+nx$ is used in both. The 1948 question is more difficult because the approximation is not given.

TABLE CXXXVb

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	Used approximation formula $(1+x)^n \doteq 1+nx$ for $(3+0.37)$	51	17.0
	Attempted to expand by the Binomial Theorem $(27+1)^{1/3}$	46	15.3
	Mechanical Errors	26	8.7
	$\sqrt[3]{28} = 3.037$ or $(3.037)^3 = 25$	33	11.0
	Error in evaluating second term of expansion	5	1.7
	$10\ (2+0.8)^{1/3}$	5	1.7
	$\sqrt[3]{30-2}$	11	3.6
	$\sqrt[3]{25+3}$	5	1.7
	Doodled etc.	33	11.0
	Incomplete	12	4.0
	Used logs to evaluate-difficulty in manipulation	4	1.3
	No Attempt	51	17.0
1944	$\{1+3(0.06)\}$	1	1.0
	$\{1+3/2(0.06)\}$	2	2.0
	Other incorrect answers	16	16.0
	No Attempt	2	2.0

XXIX a. Question53-1948 The connection between the speed V feet per sec. of a column of water produced by a pressure of head H feet, as determined experimentally, is shown in the following table.

H	4	16	25	36	64
V	15.7	32.7	39.3	49.1	63.3

Plot V against $\sqrt{H}$ and find the most probable relation expressing V in terms of H.

Correct Answer : $V = 8\sqrt{H}$ - 5 marks

b. Question 44-1944 The following table contains a few of the results obtained in an experiment.

P	3.30	4.34	5.76	7.56	10.0
h	6	7	8	9	10

Plot P/h against P and find the best fit equation expressing P in terms of h.

Correct answer: $P = \frac{5h}{(15-h)}$ - 6 marks.

TABLE CXXXVIa

Comparison of Correct Scores

Scores	1948 1.		1944 2.	
	N	%	N	%
0	70	23.3	28	28.0
1	46	15.3	26	26.0
2	81	27.0	15	15.0
3	38	12.7	19	19.0
4	30	10.0	12	12.0
5	35	11.7	0	0.0
6			0	0.0

1. Total Marks 5

2. Total Marks 6

The two questions are similar, in that information and directions are given for plotting a graph and then a formula is to be derived. As results indicate, the 1948 candidates were able to make a better showing than the 1944 candidates.

TABLE CXXXVIb

Comparison of Errors and No Attempts

Year	Explanation	N	%
1948	No Graph	27	9.0
	Confused attempt in calculating relationship	58	19.3
	Constant term too large	41	13.7
	Graph only-calculation of ralationship not attempted	27	9.0
	Assumed $V \propto \sqrt{H}$	41	13.7
	Points used in calculation too close together	3	1.0
	Points chosen were not in the table	8	2.7
	Answer in terms of x andy instead of $\sqrt{H}$ and V	21	7.0
	Mechanical Errors	19	6.3
	Plotted H against V	1	0.3
	No Attempts	19	6.3
1944	Plotted P against P/h	39	39.0
	Plotted P against h or h against P	18	18.0
	Not accurate enough	27	27.0
	Plotted correctly formula incorrect	21	21.0
	Graph only	12	12.0
	Used poor scale	8	8.0
	Mechanical Errors	6	6.0
	Incomplete	1	1.0
	Confused Attempt	1	1.0
	No Attempt	7	7.0

The following table shows the questions which were selected from the 1948 and 1944 papers for comparison.

TABLE CXXXVII

1948	1944	1948	1944
2	2	31	40a
3	11	33	5
5	4	36	28
8	8	37	29
9	10	39	38
11	3	41	19
12	11	-	-
13	7	44	39
14	12	45	9
19	16	46	30
20	17	48	41
24	21	49	42
25	18	50	24
26	22	53	44
28	23		
29	14		

The 1948 candidates were able to secure better scores than the 1944 candidates on the following 1948 questions (See table XXX for corresponding 1944 question). Number 5,8,11,12,13,20,26,28,29,31,33,37, 39,44,46,48,53. Thus of the 29 questions compared, the 1948 candidates did better on 17 of these.

Conclusions

On the basis of the comparisons, it would appear that the 1948 candidates are superior to the 1944 candidates. However, extenuating circumstanc must be considered in orderthat the comparison be valid.

(1) It must be borne in mind that the questions considered are not exactly similar. An indication has been given of this in the comments on each comparison.

(2) The 1948 Examination paper consisted of 34 multiple choice questions and 19 problem solving type questions. There is no doubt that multiple choice questions, which involve an element of recognition, are easier. The psychological effect on the pupil must be, a heartening instead of a disheartening one. The 1944 Examination paper

consisted of 25 short answer type questions. The answer and possible answers were not given. This type of question then involves an element of recall. There were 19 problem-solving type questions on the 1944 paper.

If we disregard the first 34 questions of the 1948 paper and compare results on the problem-solving portion of the paper, we note that the 1948 candidates were superior to the 1944 candidates on 6 of the 11 questions compared.

(3) During the War Years, the length of the school year was shortened to allow students to help out on the farms and in industry. The 1944 candidates then received less instruction than the 1948 candidates.

(4) Many teachers were away on war service and as a result inexperienced teachers were teaching high school. In all probability the standard of instruction in 1948 was superior to that of 1944.

(5) There were more candidates writing on the 1948 paper than on the 1944 paper.

Thus the reader can judge for himself the validity of the comparisons made. There is no doubt that results for both groups are poor and make one wonder as to the advisability of allowing students just at the matriculation level to proceed to further Academic studies in the field of mathematics.

The following Table shows a comparison of correct scores, expressed as a percent, for 1944 and 1948 candidates on various topics. All questions on the papers were used in compiling this table.

TABLE CXXXVIII

Comparison of Correct Scores[1] on Topics 1944 & 1948

Topic	1944	1948
Exponents and Logs	55	42.6
Ratio and Proportion	58	62.5
Variation	64	48.5
Functions and Graphs	45	60.6
Limits and Gradients	44	49.8
Differentiation	37	37.5
Integration	30.3	4,8
Arithmetic Progression	57	80.3
Maxima-Minima	32	39
Binomial Theorem	41	45.9
Permutations and Combinations	56	33.1
Geometric Progression	28	47.0
Annuities	32	38.2
Empirical Formulae	40	11.7

1. Expressed as a % of total possible mark.

CHAPTER VII

I Text Book Analysis

The text book used in this course - Durell and Wright "Senior Algebra" edited by Dr. E. W. Sheldon and D. L. Shortliffe, has been in use since the fall of 1933. In view of modern trends in text books there is no doubt but that the text book should be revised. For example, there should be more practise exercises on the fundamentals of calculus e.g. integration. Then following this practise on simple integration should some the differential equations. The practise exercises should be simple. It is the writers belief that a great many of the errors made in integration are due primarily to insufficient practise. Again the section dealing with Annuities should have many more short practise exercises.

A. Analysis of the Text for Relationships

1. If $\frac{a}{b} = \frac{d}{d} = \frac{e}{f} = \ldots$

then each fraction $= \frac{pa + qc + re + \ldots}{pb + qd + rf + \ldots}$

$= \frac{n(pa + qc + re + \ldots)}{n(pb + qd + rf + \ldots)}$

2. The Statement the y varies as x may be expressed as :

a. $y \propto x^n$

b. $\frac{y_p}{y_q} = \frac{x_p^n}{x_q^n}$ where x_p, y_p are any pairs of corresponding values of x, y and x_q, y_q any other pair.

c. $\frac{y_1}{x_1^n} = \frac{y_2}{x_2^n} = \frac{y_3}{x_3^n} = \ldots\ldots = k$

d. If x is altered in any ratio $\lambda:1$ then y is altered in the ratio $\lambda^n:1$

e. The result of plotting y against x^n gives a straight line through the origin.

3. a. If x varies directly as the p^{th} power of y and inversely as the q^{th} power of z then $x = k \frac{y^p}{z^q}$ where k is a constant independent of x,y,z.

If $x = a$ when $y = b$ and $z = c$ then $k = \frac{c^q a}{b^p}$ or $x = \frac{ac^q}{b^p} \cdot \frac{y^p}{z^q}$

b. If y is altered in the ratio $\lambda:1$, and if z is altered in the ratio $\mu:1$, then x is altered in the ratio $\frac{\lambda^p}{\mu^q}:1$.

4. The function $f(x) \equiv \frac{a(x-b)}{c(x-d)}$, vanishes when $x = b$ and is not defined when $x = d$.

5. Any expression whose value can be determined when the value of x is known, can be represented by the symbol $f(x)$, which is shorthand for "a function of x ".

6. $\underset{n\to\infty}{Lt.}\ (x+1/n) = x$ or $\lim_{n\to\infty}(x+1/n) = x$.

7. If a function of x is represented by a graph, the rate at which the function is increasing for any value of x is represented by the slope of the tangent to the graph at the point corresponding to that value of x, provided that units used in drawing the graph are taken into account.

8. It is possible to have a function of x to which for one or more special values of x, no meaning can be given.

9. The equation of the straight line through (o,b) having gradient m is $y = mx + b$.

10. The equation of the straight line through (x,y) having gradient m is $y-y_1 = m\ (x-x_1)$

11. The average gradient of a graph over an interval is $\frac{f(x+h) - f(x)}{h}$, providing the interval is small.

12. The gradient at a point on a graph is given by $\underset{h\to 0}{Lt.}\ \frac{f(x+h) - f(x)}{h}$, prodiding the interval is small.

13. The gradient of the graph at a point is the gradient of the tangent at the point, to the curve.

14. $\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$

15. $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$

16. $\frac{d}{dx}(x^n) = nx^{n-1}$, where n is integral or fractional, positive or negative.

17. $\frac{d}{dx}(c) = 0$

18. $\frac{d}{dx}(c\,x^n) = c\,\frac{d}{dx}(x^n) = c\,nx^{n-1}$

19. $\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}\left[f(x)\right] \pm \frac{d}{dx}\left[g(x)\right]$

20. For the function $y = f(x)$

a. If $\frac{dy}{dx} = 0$ and changes sign, there is a maximum or minimum.

b. If $\frac{dy}{dx}$ changes from + to -, there is a maximum.

c. If $\frac{dy}{dx}$ changes from - to +, there is a minimum.

d. If $\frac{dy}{dx} = 0$ but does not change sign, there is neither a maximum nor a minimum.

21. An approximate value for $f(x+\Delta x) - f(x)$ is $\frac{d}{dx}(f(x)) \times \Delta x$.

22. $\int f(x)\,dx$ represents the function which when differentiated with respect to x give $f(x)$.

23. Any Arithmetical Progression can be written

in the form a, a+d, a+2d, ...

a. The n^{th} term = a + (n-1)d, or

l = a + (n-1)d where l is the last term.

b. The sum of n terms: S = n X average of the terms.

24. Any Geometrical Progression can be written in the form: a, ar, ar^2, ar^3, ...

a. The n^{th} term is ar^{n-1}

b. The sum of n terms= $\frac{a\,(r^n-1)}{r-1} = \frac{a\,(1-r^n)}{1-r}$

c. The sum to infinity = $\frac{a}{1-r}$.

25. Combinations refers to things chosen without regard to their order.

26. Permutations refers both to the things chosen and their order.

27. $(x+a)^n \equiv x^n + nx^{n-1}a + \frac{n(n-1)}{1.2}x^{n-2}a^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1.2\ldots\ldots r}x^{n-r}a^r + \ldots\ldots + a^n$

28. If the graph obtained by plotting y against x is, or approximates to, a straight line, and if (x_1,y_1), (x_2y_2) are two points on it, preferably at opposite ends, then y can be expressed as a function of x by the equation: $\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$.

29. If the graph obtained by plotting y against x is a curve, it may be possible to obtain a straight line by plotting:

a. log y against log x.

b. log y against x.

c. y against log x.

d. y against 1/x, etc.

30. Where the data are experimental, coeffieients in the functional relationship should be expressed as decimals and must not be given to more significant figures than the data and the method justify.

B Analysis of the Text Book for Concepts

I Vocabulary

The following list contains all the technical terms or vocabulary in the text.

CHAPTER I

p. 1 Ratio
Proportion
Variation
Ratio-relation

p. 2 Value

P. 3 Ratio form
divided externally
divided internally

p. 5 Approx. value of a ratio

p. 6 Fourth proportional
Third proportional
Mean proportional
Continued proportion
ratio equality

p. 7. Geometrical interpretation

P. 8 is directly proportional to

p. 9 varies directly as; inversely proportional to;
varies inversely as

p.15 variation relation

p.16 constant

p.17 Joint variation

CHAPTER II

p. 23 functions of one variable
representation of functions
known function
sketch

p. 24 increases without limit
approaches zero as a limit
decreases without limit

p. 25 range of values
positive function
negative function
absolute value

p. 27 defined for all values
undefined

p. 29 construction of functions

p. 31 Graphical solutions

p. 33 notation
solve by algebra
maximum area or value

p. 34. positive value

p. 35 minimum value

p. 39 real numbers
discriminant
real and distinct
real and coincident
imaginary

p. 41 functional notation
a function of x - $f(x)$

CHAPTER III

p. 44 Limits and gradients
positive integral values

p. 45 function increases
function decreases

p. 47 o/o meaningless expression
limit not attained

p. 50 Rates of change from statistics
Rates of change from formulae

p. 55 rate of increase
average gradient

p. 56 gradient of the curve
gradient of the tangent

p. 60 linear function

p. 61 functions in general
average gradient over an interval

CHAPTER IV

p. 62 Differentiation

p. 63. gradient of the secant
differential coefficient $\frac{dy}{dx}$
derivative
derived function
differentiating with respect to x

p. 64 Successive differentiation

p. 70 turning points
increasing function
decreasing function
turning value
maximum
minimum

p. 75 maxima
minima

p. 78 approximations and errors

p. 83 parabolas

p. 84 cubic parabolas
power functions
hyperbolas

CHAPTER V

p. 85 Integration
general solution
differential equation

p. 86 solving or integrating

p. 89 area bounded by a curve

p. 91 indefinite integral
definite integral

p. 93 rotating a part of the graph

CHAPTER VI

p. 96 Sequence
series
term
n th term

p. 99. arithmetical progressions
common difference
arithmetic mean

p.102 Geometrical progressions
common ratio
geometric means

p.103 Index notation
exponent

p.105 Infinite series
sum to infinity of a series

p.108 Growth functions

p. 110 annuities
worth
present value
cash value
future value
compounded annually

p. 111 discounting
cash value of an annuity

p. 112 deferred
cash value of bond redemption

p. 113 accumulated value of an annuity left on deposi
payee
maturity

p.114 depreciation fund
annual payment

p. 115 perpetuities
capitalization

p. 116 sinking fund

CHAPTER VII

p. 118 permutations
combinations
compound events

p. 122 ${}^{n}P_{r}$

p. 124 Combinations ${}^{n}C_{r}$

CHAPTER VIII

p. 129 Binomial Theorem

p. 131 Coefficient

p. 132 Approximations
Binomial Expansion

CHAPTER IX

p. 140 Empirical Formulae
plot
against

p. 148 logarithm

II The following are the definitions which the text book gives for the technical terms:

(1) The ratio of two numbers a, b is measured by the fraction a/b and is often written a:b

(2) If a/b = c/d, the numbers a, b, c, d are said to be in proportion.

(3) If a/b = c/d, then d is called the fourth proportional to a, b, c.

(4) If a/b = b/c, then C is called the third proportional to a, b.

(5) If a/b = b/c, then b is called the mean proportional between a, b.

(6) If a/b = b/c = c/d = d/e = etc, a,b,c,d,e are said to be in continued proportion.

(7) The ratio equality x/d = y/b = z/c = w/d = ... can also be written in the for x: y: z: w: ... = a: b: c: d: .....

(8) If $x_1 : x_2 : x_3 : \ldots = y_1 : y_2 : y_3 : \ldots$, then x is said to be directly proportional to y.

(9) y is said to be a function of x, if y can be determined when x is known.

(10) If $y_1 : y_2 : y_3 : \ldots = x_1 : x_2 : x_3 : \ldots$, then y is said to vary directly as x or simply vary as x ($y \propto x$).

(11) If $y_1 : y_2 : y_3 : \ldots = x_1^n : x_2^n : x_3^n : \ldots$, then y is said to vary directly as the n^{th} power of x ($y \propto x^n$)

(12) If $y_1 : y_2 : y_3 : \ldots = \frac{1}{x_1^n} : \frac{1}{x_2^n} : \frac{1}{x_3^n}$ then y is said to vary inversely as the n^{th} power of or $y \propto \frac{1}{x^n}$.

(13) If x varies directly as the p^{th} power of y and inversely as the q^{th} power of z then $x \propto \frac{y^p}{z^q}$

(14) The function $f(x) \equiv \frac{a(x-b)}{c(x-d)}$ vanishes when x = b and is undefined when x = d.

(15) $f(x) \to +\infty$ when $x \to p$: a $f(x)$ can be made to exceed any number however large if x is given a value sufficiently close to p or any value closer than this to p.

(16) The expression $b^2 - 4ac$ is called the discriminant of the equation.

(17) Whenever $b^2 - 4ac > 0$, the roots of the equation are real numbers.

(18) Whenever $b^2 - 4ac = 0$ the equation has one real root.

(19) Whenever $b^2 - 4ac < 0$ the equation has two imaginary roots

(20) A function of x - $f(x)$

(21) Lt. or $\lim_{n \to \infty} (1 + 1/n) = 1$ The limit as n increases without limit of $(1+1/n) = 1$.

(22) It is possible to have a $f(x)$ to which, for one or more special values of x, no meaning can be given. e.g. $\frac{x^2-1}{x-1}$ has no meaning when x = 1

(23) The gradient of the straight line through (x_1, y_1) and $(x_2 y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$

(24) The average gradient of the straight line of the straight line y = mx + b over any interval is m; its gradient at any point is m.

(25) The average gradient of a curve over an interval is given by $\frac{f(x+h) - f(x)}{h}$

(26) The gradient of the curve at a point is given by $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

(27) The expression $\underset{\Delta x \to o}{\text{Lt.}} \frac{\Delta y}{\Delta x}$ is written $\frac{dy}{dx}$ or $\frac{d}{dx}(y)$ and is called the differential coefficient of y with respect to x.

(28) The expression $\underset{x \to o}{\text{Lt.}} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ is written $\frac{d\,f(x)}{dx}$ or $\frac{d}{dx} f(x)$ and is called the differential coefficient of $f(x)$ with respect to x, or the derivative of $f(x)$ or the derived function of $f(x)$.

(29) The process of finding the limit, as expressed in (28) above, is called differentiating with respect to x.

(30) $\frac{d^2y}{dx^2}$ is called the second differential coefficient of y with rexpect to x.

(31) A function $f(x)$ is called an increasing function of x for any range of values of x in which, as x increases the value of $f(x)$ also increases.

(32) A function $f(x)$ is called a decreasing function of x for any range of values of x in which, as x increases, the value of the $f(x)$ decreases.

(33) Turning point or separating point.

(34) Turning value is the value of $f(x)$ corresponding to the separating point.

(35) a. If $\frac{dy}{dx} = o$ and changes sign from + to - then the function is said to pass through a maximum.

b. If $\frac{dy}{dx} = o$ and changes sign from - to + then the function is said to pass through a minimum.

(36) An approximate value for $f(x+\Delta x)-f(x)$ is $\frac{d}{dx} f(x) \times \Delta x$

(37) If in calculating the value of $f(x)$, there is a small error Δx in the value substituted for x, the error in the result is approximately $\frac{d}{dx} f(x) \times \Delta x$.

(38) If y is a function of x and x is a function of t, the rate at which y changes is given by $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

(39) Functions of the form ax^2+bx+c are called quadratic functions.

(40) Functions of the form ax^3 are called cubic functions.

(41) Functions of the form ax^n are called power functions.

(42) An equation involving $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ is called a differential equation.

(43) The process of expressing y as a function of x is called solving or integrating the differential equation.

(44) In the equation $y = 1/3\ x^3 + 7x+C$, where C is any constant, satisfies the given equation and is called the general solution;

(45) $\frac{dy}{dx} = f(x)$ is usually express in the form $y = \int f(x)\ dx$

(46) $\int_a^b f(x)\ dx$ is used to express the value of $\int f(x)\ dx$ when $x = b$ minus the value of $\int f(x)\ dx$ when $x=a$

(47) The expression $\int f(x)\ dx$ is called an indefinite integral because its value contains an arbitrary constant.

(48) The expression $\int_a^b f(x)dx$ is called a definite integral because the arbitrary constant of integration disappears automatically.

(49) Numbers following each other, usually in a pattern are spoken of as a sequence.

(50) Series is a term used in reference to a sum.

(51) A sequence of numbers in which each term exceeds the preceding term by the same amount, positive or negative is called an Arithmetic Progression

(52) The amount by which each term exceeds the preceding terms either negative or positive is called the common difference.

(53) If three numbers are in A. P., the middle number is called the Arithmetic mean of the other two.

(54) A sequence of numbers in which each term bears a fixed ratio to the term preceding it is called a Geometric Progression.

(55) The fixed ratio which each term bears to the term preceding it is called the common ratio.

(56) If three numbers are in G. P., the middle number is called the Geometric mean of the other two.

(57) It is customary to call an endless series an infinite series.

(58) Where there is a limit of the sum of n terms of a series, the limit is called "the sum to infinity" of the series.

(59) Such a function as $(1.03)^x$ is called a growth function because it represents the magnitude of a quantity which increases in such a way that its growth over a definite period is proportional to its magnitude at the beginning.

(60) $ x at 5% per annum compounded annually, amounts to $x $(1.05)^2$ at the end of two years. Here the $x is called the present or cash value of $x $(1.05)^2$ due in two years.

(61) To find the future sum that corresponds to a give cash sum, multiply by the amount of $1.

(62) The process of finding the cash value of a future sum of money is called discounting the future sum.

(63) An annuity is a succession of equal sums of money due at equal intervals of time.

(64) A perpetuity is an annuity that has an unlimited time to run.

(65) When an anual income or outgo that is regarded as a perpetuity is discounted, it is said to be capitalised.

(66) Permutations involve an arrangement or order.

(67) Combinations involve a choice of groups with no arrangement or order considered.

(68) $(x+a)^n \equiv x^n + nx^{n-1}a + \frac{n(n-1)}{1.2} x^{n-2} a^2 + \ldots\ldots$

$+ \frac{n(n-2)(n-2)\ldots(n-r+1)}{1.2.3\ldots\ldots r} x^{n-r} a^r + \ldots\ldots + a^n$

This result is called the Binomial Theorem.

(69) If results, experimentally arrived at seem to follow a certain law, then this law is called an Empirical Formula.

C Analysis of Text Book for Techniques or Routines

The following are the methods which are outlined in the text for the solution of problems in certain topics. These with practise become the tools of the course.

1. Finding the third, fourth and mean proportion.
2. Simplifying a ratio.
3. Finding the value of a ratio.
4. Expressing ratios in simplest forms.
5. Proving equality of Algebraic ratios (k method).
6. Finding the constant in direct, inverse and joint variation.
7. Expressing the direct, inverse and joint variation relationships.

8. Graphical representation of a variation relationship.

9. Sketching graphs of functions of one variable.

   a. x positive

   b. x large and positive

   c. x small and positive

   d. x = o

   e. x negative etc.

10. Construction of functions.

11. Graphical solutions of functions-range of x.

12. Solution of a function by algebra.

13. Application of functional notation.

14. Finding limits.

15. Finding gradient of a straight line through two points.

16. Finding equation through one or two points.

17. Finding average gradient of a curve over an interval.

18. Finding the gradient of a curve at a point.

19. Differentiating by using delta notation.

20. a. $\frac{d}{dx}(x^n) = nx^{n-1}$ b. $\frac{d}{dx}(C) = o$

    c. $\frac{d}{dx} Cx^n = Cnx^{n-1}$ d. $\frac{d}{dx}\left[f(x) \pm g(x)\right] =$

    $\left[\frac{d}{dx} f(x)\right] \pm \left[\frac{d}{dx} g(x)\right]$

21. Finding the second differential coefficient.

22. Application of criteria for turning points.

23. Application of differentiation - maxima-minima.

24. Definite and indefinite integration.

25. Approximations and finding errors.

26. Finding areas and volumes by integration.

27. Finding the n^{th} term of a series.

28. Summing an Arithmetic Series.

29. Finding the n^{th} term of an Arithmetic Progresion.

30. Summing a Geometrical Progression.

31. Finding the Arithmetic Mean.

32. Finding the n^{th} term of a G. P.

33. Finding the sum to infinity of a G.P. $(-1 \leqq r \leqq 1)$

34. Finding the Geometric Mean.

35. Finding the number of terms in a G. P.

36. Solving growth functions.

37. Finding future value of a sum of money.

38. Finding cash value of a sum of money.

39. Discounting a future sum of money.

40. Finding cash value of an annuity.

41. Finding accumulated vlue of an annuity left on deposit.

42. Finding the annual payment.

43. Summing perpetuities.

44. Solving depreciation problems.

45. Capitalising a sum of money.

46. Finding the number of ways a group of articles can be selected and arranged in a row (Permutation)

47. Finding the number of ways a group of articles can be chosen from a given number of articles. (Combinations)

48. Expanding a binomial according to the theorem.

49. Using the Binomial expansion for approximations.

50. Finding the best fit line for a series of points by observation.

51. Finding the equation of the best fit line.

52. Application of Logarithms.

II Analysis of the Examination Paper

The following section deals with the analysis of the 1948 Examination paper. The analysis is made on the same basis as that on which the analysis of the text book was made. It is reasonable to expect that not all of the routines or concepts contained in the text book can be tested on a paper requiring three hours for completion. In each instance the question involving the relationship, concept or technique will be given together with the mark alloted. For example in order to solve question 9, a candidate must know the meaning of the word "undefined". This word appears as part of the vocabulary for the course. Therefore two marks have been allotted to this word on the basis of the candidate's knowledge of the word. Again, question 48 requires that the candidate find the cash value of an annuity at maturity. This is a routine which the candidate has learned. Therefore four marks have been allotted for this routine. Three Three marks have been allotted to Geometric Series under Relationships because the candidate must also have a knowledge of Geometric Series in order to solve the problem. In the same manner marks have been distributed under vocabulary in the following manner

for question 48: a. annuity - 1 mark. b. accumulates - 1 mark. c. compounded annually - 1 mark. d. maturity 1 mark. e. payee - 1 mark. An attempt has been made to allot marks in each case on the basis of the marks allotted for correct solutions as set down by the Examinations Board. Thus in no instance will the total marks given for the paper (153) be the total given for knowledge of relationships, or routines, or concepts.

TABLE CXXXIX

A Analysis of Examination Paper for Relationships

1	1. Relationships dealing with ratio and proportion	Questions # 2,3,4,35 10 marks
2	2. Relationships dealing with direct variation	Questions# 11,36 4 marks
3	3. Relationship dealing with inverse variation	Questions# 36 2 marks
8 4 6	4, Relationships dealing with limits	Question # 9 2 marks
5	5. Relationships dealing with functions and graphs	Questions# 5,6,7,8,10, 13,21,32,33, 34,37,38,40, 45,39,51 43marks
7 20	6. Relationships dealing with maximum and minimum	Questions # 22,39,51 4 marks
9 10	7. Relationships dealing with gradients	Questions # 12,40 4 marks
11	8. Relationships dealing with average gradients	Question # 42b 1 mark
12 13	9. Relationships dealing with gradients at a point	Question # 42c 2 mark
15 16 17 18 19	10. Relationships dealing with differentiation	Questions# 14,18,29, 39,42a,43 51 15 mark

*21	11.	Relationships dealing with approximation	Question # 29 2 marks
22	12.	Relationships dealing with integration	Questions# 19,44,52 13 marks
23	13.	Relationships dealing with Arithmetic Progressions	Questions# 20,47 5 marks
24	14.	Relationships dealing with Geometric Progressions	Questions# 25,31,46, 48 10 marks
25	15.	Relationships dealing with Permutations & Combinations	Questions# 24,26,49 7 marks
27	16.	Relationships dealing with Binomial Theorem	Questions# 23,28,50 8 marks
28 29 30	17.	Relationships dealing with Empirical Formula	Questions# 53 5 marks
			TOTAL 137 marks

*Relationship as numbered in Chapter VIII I A

TABLE CXL

Marks Allocated to Different Questions on Basis of Relationship Contained

#	Mark	#	Mark	#	Mark	#	Mark
1	0	15		29	2	43	5
2	2	16		30		44	6
3	2	14		31	2	45	5
4	2	18	2	32	2	46	3
5	2	19	2	33	2	47	3
6	2	20	2	34	2	48	3
7	2	21	2	35	4	49	3
8	2	22	2	36	4	50	4
9	2	23	2	37	6	51	3
10	2	24	2	38	3	52	5
11	2	25	2	39	1	53	5
12	2	26	2	40	3		
13	2	27		41			
14	2	28	2	42	5		

TABLE CXLI

B Analysis of Examination Paper for Vocabulary

Technical Term	Question	# Marks	Total
Formula	7	2	2
Undefined	1 9	2 2	4
Fourth Proportional	2	2	2
Ratio	3	2	2
Are to each other	4	2	2
Graph of the function	5 40	2 3	5
Function	6 13 22 32 33	2 2 2 2 2	10
$f(x)$ - Notation	8 13 42	2 2 5	9
Graph of the equation	10 21	2 2	4
Varies directly	11	2	2
Gradient	12 40	2 2	4
$\frac{dy}{dx}$	14 42c 43	2 2 1	5
Logarithm	16 30	2 2	4

TABLE CXLI
continued

$\frac{d^2y}{dx^2}$	18	2	2
$\int_4^9$	19	2	2
Arithmetic Progression	20	2	2
Maximum	22 39 51	2 2 4	
Positive integral values	23	2	2
Expansion	23	2	2
Combinations	26	2	2
Geometric Series	25 31 46	2 2 2	 6
Terms	25 20 31 23 46 23 12 36	2 2 2 2 1 2 2 1	 12
Ratio (G. P.)	25 46	2 1	 3
Permutations	24	2	2
Depreciates	27	2	2
Worth	27	2	2
Coefficient	28	2	2
Error in approximation	29	2	2
Sum to infinity	31	2	2

TABLE CXLI CONTINUED

Positive value	35	2	2
Varies jointly	36	2	2
Varies inversely	36	2	2
Sketch graph of a function	37	6	6
Sketch a graph	38	3	3
$\to 2$ when $x \to +\infty$ $\to +\infty$ " $x \to o$ provided $x > o$ $\to -\infty$ " $x \to o$ $\to -2$ " $x \to -\infty$	38	3	3
Δy	42a	2	2
$\frac{\Delta y}{\Delta x}$	42b	1	1
Rotating part of graph	44	1	1
Annuity	48	1	1
Accumulates	48	1	1
Compounded annually	48	1	1
Maturity	48	1	1
Payee	48	1	1
Binomial Theorem	50	3	3
Area bounded by a curve	52	2	2
Plot	53	2	2
Most probable relation	53	1	1
Against	53	2	2
Exponent	23 17	2 2	4
Geometrical Interpretation	42b	2	2
Solve the equation	45	5	5

TOTAL MARKS 156

TABLE CXLII

C Analysis of Examination Paper for Routines

Routine	Question#	Marks	Total
Finding a fourth proportional	2	2	2
Finding the ratio having the greatest value	3 35	2 4	6
Expressing a direct variation relationship	4	2	2
Solution of a function by Algebra	7 10 21 45	2 2 2 5	11
Construction of functions	5 53	2 2	4
Expressing a functional relationship	6 32 33 34	2 2 2 2	8
Application of functional notation	8 13	2 2	4
Finding limits	9	2	2
Finding the constant in direct, inverse and joint variation	11 36	2 4	6
Finding the gradient of a straight line through 2 points	12	2	2
Differentiation according to rule	14 43	2 5	7
Applying logarithms	16 30 41	2 2 4	8

TABLE CXLII CONT'D.

Find the second differential	18	2	2
Definite Integration	19	2	2
Finding the n^{th} term of an A.P.	20	2	2
Finding the maximum value of a function	22 39 51	2 1 1	4
Expansion of a Binomial	23 28 50	2 2 4	8
Permutations	24 49	2 2	4
Finding number of terms in a G. P.	25 46	2 3	5
Combinations	26 49	2 2	4
Finding value after depreciation	27	2	2
Approximations	29	2	2
Find the correct Geometric Series to fit a given set of data	31	2	2
Sketching the graph of a function of one variable	37 38 40	6 3 5	14
Finding volumes by integration	44	6	6
Finding areas by integration	52	5	5

TABLE CXLII CONT'D

Finding the accumulated value of an annuity bearing no interest	47	3	3
Finding the value of an annuity at maturity	48	4	4
Finding the equation of a best fit line	53	3	3
Finding the value of x^0	1	2	2
Using 10^{-x} notation	17	2	2

TOTAL 138

CHAPTER VIII

Pupils' Mastery of Relationships, Concepts and Routines

This chapter gives an indication of howwell each relationship, concept and routine has been mastered by the candidates in light of the correct and partial scores which they were able to secure. For example, Relationship 1 deals with a knowledge of ratio and proportion. Questions 2,3,4 and 35 of the examination paper have been designed with a view of testing candidates knowledge of relationships involving ratio and proportion. Question 2 deals with fourth proportionals; question 4 with finding a ratio and question 35 with ratio inequalities. These four questions have a total score of 10 marks allocated to them. Thus for 300 papers the total mark would be 3000 i.e. if each candidate was able to secure perfect scores for each question. The following table indicates the scores which the candidates made on each question dealing with this relationship.

TABLE CXLIII

Total Correct and Actual Scores-Ratio & Proportion

Question #	Mark Allocated	Total Correct Score 1.	Actual Score 2.
2	2	600	488
3	2	600	400
4	2	600	544
35	4	1200	39
TOTALS	10	3000	1071

1.2. - for 300 papers.

Thus it can readily be seen that pupil mastery of this relationship is $\frac{1071}{3000} \times 100 = 35.7\%$

Each relationship, Concept and Routine will be treated in a similar manner.

A-Pupil Mastery of Relationships

TABLE CXLIV

Relationships	Text emphasis	% of Exam	Pupil Mastery -%
1. Relationships dealing with ratio and proportion	8 pages	7.3	35.7
2. Relationships dealing with direct variation	9 "	2.8	79.0
3. Relationships dealing with joint variation	6 "	1.5	63.5
4. Relationship dealing with limits	7 "	1.5	9.7
5. Relationships dealing with functions and graphs	21 "	31.4	58.5
6. Relationships dealings with maximum and minimum	8 "	2.9	47.0
7. Relationships dealing with gradients and average gradients	8 "	3.6	59.8
8. Relationship dealing with gradient at a point	2 "	1.5	26.5
9. Relationships dealing with differentiation	8 "	10.9	49.1
10.Relationships dealing with approximations	4 "	1.5	36.6
11.Relationships dealing with Integration	10 "	9.6	23.0

cont'd.

TABLE CXLIV cont'd.

12.Relationships dealing with Arithmetic Progression	4 pages	3.6	58.3
13.Relationships dealings with Geometric Progression	4 "	7.5	40.9
14.Relationships dealing with Permutations and Combinations	10 "	5.1	28.5
15. Relationships dealing with Binomial Theorem	10 "	5.7	41.3
16. Relationships dealings with Empirical Formulae	8 "	3.6	41.1
		100.0	43.6 Average

Thus, from this table, it is evident that the candidates in this group have a 43.6% knowledge of the Relationships of the course.

B-Pupil Mastery of Vocabulary & Technical Terms

TABLE CXLV

Technical Term	% of Exam	Pupil Mastery-%
Formula	1.3	92.3
Undefined	2.6	60.0
Fourth Proportional	1.3	81.3
Ratio	1.3	66.6
Are to each other	1.3	75.6
Graph of the function	3.2	72.2
Function	6.4	57.6
$f(x)$ - (notation)	5.7	60.3
Graph of the equation	2.6	65.1
Varies directly	1.3	94.6
Gradient	2.6	65.1
$\frac{dy}{dx}$	3.2	63.7
Logarithm	2.6	25.2
$\frac{d^2y}{dx^2}$	1.3	94.3
$\int_4^9$	1.3	48.6
Arithmetic Progression	1.3	80.3
Maximum	5.2	41.2
Positive Integral values	1.3	47.6
Expansion	1.3	47.6

cont'd.

TABLE CXLV cont'd.

Combinations	1.3	62.0
Geometric Series	3.9	54.3
Terms	7.8	73.2
Ratio-(Geometric Progression)	1.9	74.1
Permutations	1.3	36.3
Depreciates	1.3	46.6
Worth	1.3	46.6
Coefficient	1.3	73.6
Error in Approximation	1.3	36.6
Sum to Infinity	1.3	71.6
Positive value	1.3	4.3
Varies jointly	1.3	63.5
Varies inversely	1.3	63.5
Sketch graph of a function	3.9	63.0
Sketch graph	2.0	49.7
$\to 2$ when $x \to +\infty$ $\to +\infty$ " $x \to o$ provided $x > o$ $\to -\infty$ " $x \to o$ $\to -2$ " $x \to -\infty$	2.0	49.7
Δy	1.3	24.0
$\frac{\Delta y}{\Delta x}$	0.7	38.6
Rotating part of a graph	0.7	49.3
Annuity	0.7	46.6
Accumulates	0.7	46.6

TABLE CXLV Concluded

Compounded annually	0.7	46.6
Maturity	0.7	46.6
Payee	0.7	46.6
Binomial Theorem	2.0	36.0
Area bounded by a curve	1.3	14.5
Plot	1.3	69.0
Most probable relation	0.7	76.6
Against	1.3	69.0
Exponent	2.6	45.2
Geometrical Interpretation	1.3	38.6
Solve the equation	3.2	13.5
	AVERAGE	53.3

Candidates in this group have a 53.3% mastery of the Technical terms and vocabulary of the course.

C-Pupil Mastery of Techniques or Routines

TABLE CXLVI

Routine or Technique	% of Exam	Pupil Mastery-%
Finding a fourth proportional	1.5	81.3
Finding the ratio having the greatest value	4.5	36.6
Expressing a direct variation relationship	1.5	90.6
Solution of an equation by algebra	7.9	51.4
Construction of functions	3.0	74.8
Expressing a functional relationship	6.0	59.4
Application of a functional notation	3.0	92.7
Finding limits	1.5	19.3
Finding the constant in direct inverse and joint variation.	4.5	68.1
Finding the gradient of a straight line through 2 points	1.5	81.3
Differentiation according to formula	5.1	44.3
Use of Logarithms	6.0	31.8
Finding the second differential	1.5	94.3
Solving problems involving definite integration (limits given)	1.5	48.3
Finding the n^{th} term of an A. P.	1.5	80.3

cont'd.

TABLE CXLVI concluded

Expanding a Binomial and miscellaneous problems dealing with the Binomial Theorem e.g. Approximation	6.0	41.3
Finding the maximum value of a function.	3.0	47.0
Solution of problems involving permutations (answer given)	1.5	36.3
Solution of problems involving permutations (no answer given)	1.5	1.0
Solution of problems involving combinations (Answer given)	1.5	62.0
Solution of problems involving combinations (No answer given)	1.5	1.0
Approximation routines	1.5	36.6
Finding the correct geometric Series to fit given data	1.5	71.6
Sketching graphs	10.1	45.0
Finding volumes by integration	4.5	27.1
Finding areas by integration	3.4	8.0
Finding the value of an annuity bearing interest	3.0	23.7
Finding the equation of a best fit line.	2.2	46.0
Finding the value of x^o	1.5	84.0
Using 10^{-x} notation	1.5	43.0

AVERAGE 50.7

Candidates in this group have a 50.7% mastery of the Routines and Techniques of the course.

CHAPTER IX

Pupil Mastery of the Course on the Basis of Topics

The course has been divided up into broad divisions or topics as follows:

1. Exponents and Logarithms
2. Ratio and Proportion
3. Functions and Graphs
4. Variation
5. Limits and Graphs
6. Diferentiation
7. Integration
8. Series and Sequences
9. Permutations and Combinations
10. Binomial Theorem
11. Annuities
12. Empirical Formulae.

The following table shows the questions of the Examination paper which deal with each of the topics and also the total marks for these questions.

TABLE CXLVII

Topic	Question Numbers	Marks	% of Total Marks
Exponents & logs	1,15,16, 17,30,41	14	9.1
Ratio & Proportion	2,3,4,35	10	6.5
Functions & Graphs	5,6,7,8,10, 13,21,32,33, 34,37,38,40 45	39	25.5
Variation	11,36	6	3.9
Limits & Gradients	9,12	4	2.7
Differentiation	14,18,22,29 39,42,43,51	29	18.9
Integration	19,44,52	13	8.6
Series & Sequences	20,25,31,46	9	5.9
Permutations & Combinations	24,26,49	7	4.6
Binomial Theorem	23,28,50	8	5.2
Annuities	27,47,48	9	5.8
Empirical Formulae	53	5	3.3
		153	100.0

The following table shows how well, on the basis of correct and partial scores, the candidates were able to do on each topic. The same method has been used here in arriving at percentages as was used in Chapter IX.

TABLE CXLVIII

Topic	Mark Allocated	Average Score made by Candidate	% of total Possible Score
Exponents & Logs	14	6.6	46.8
Ratio & Proportion	10	4.9	49.0
Functions & Graphs	39	21.5	55.1
Variation	6	4.1	68.0
Limits & Gradients	4	2.0	50.3
Differentiation	29	10.9	37.7
Integration	13	3.0	23.0
Series and Sequences	9	5.1	56.4
Permutations and Combinations	7	2.0	28.5
Binomial Theorem	8	3.3	41.3
Annuities	9	3.9	42.8
Empirical Formulae	5	2.1	41.1
TOTAL	153	TOTAL 69.4	AVG.45.0

CHAPTER X

Summary and Conclusions

I Validity of the Test

As has already been noted, student success in the various test items varied greatly. Some questions were extremely easy as for example question 8 for which 95.3% of the candidates were able to secure a perfect score. In contrast to this is question 39 where only 0.7% of the candidates were able to secure a perfect score.

Examination items can be classified as extremely easy, easy, relatively easy, difficult etc. However, many factors enter into the setting up of criteria for such a classification. Because of individual differences, no two pupils will find the same question easy.

The classification which follows is somewhat arbitrary. It should also be remembered that these students are at the minimum matriculation level.

In the table which follows, the criterion adopted has been whether 70% or more of the candidates have been able to secure perfect scores. These will be called the Easier items.

TABLE CXLIV

The Easier Items

Question	Topic	% Perfect Scores
1	Exponents	84
2	Proportion	81.3
4	Ratio	90.7
5	Functions and graphs	80.7
7	Formula transformation	92.4
8	Functional Notation	95.3
10	Functions and graphs	84.3
11	Variation	94.6
12	Gradients	81.3
13	Functional Notation	90.0
14	Simple differentiation	94.3
15	Algebraic fractions	73.3
18	Successive differentiation	94.4
20	Arithmetic progression	80.3
21	Functions and graphs	72.0
28	Binomial Expansion	73.7
31	Infinite G. P.	71.7
34	Arithmetical manipulation	79.3

Thus it may be noted that the easier items were all from the multiple choice part of the paper. They comprise slightly more than ½ of the total number of questions on this part of the paper. It is interesting to note that no questions appear in the above list from the second half of the paper (questions 35-53).

In the table which follows, the criterion adopted has been whether 50%-69% of the candidates have been able to secure perfect scores. These will be called the relatively difficult items.

TABLE CL

Relatively Difficult Items

Question	Topic	% Perfect Scores
3	Ratios	66.7
26	Combinations	62.0
33	Functional relationship	66.0

Again, no questions appear in this list from second half of the paper.

In the following table, the criterion adopted has been whether 35%-50% of the candidates have been able to secure perfect scores. They will be called the difficult items.

TABLE CLI

Difficult Items

Question	Topic	% Perfect Scores
6	Functional relationship	46.7
16	Logs	43.7
17	10^{-x} notation	43.0
19	Definite Integration	48.3
22	Maximum	39.0
23	Binomial Theorem	47.3
24	Permutations	36.3
25	Geometric Series	48.0
27	Depreciation	46.7
36	Joint Variation	43.3
38	Graph Sketching	36.3
42b	Defining $\frac{\Delta y}{\Delta x}$	38.7
47	Annuities - no interest	35.4

Questions 36,38,42b and 47 are from the second part of the paper and are classified as difficult items for pupils of this group.

The following table shows the items which were found to be extremely difficult by this group.

TABLE CLII

Extremely Difficult Items

Question #	Topic	% Perfect Score
9	Limits	19.3
30	Logarithmic Manipulation	6.7
35	Ratios of inequalities	2.0
37	Graph sketching	30.7
39	Maxima-Minima problem of volume	0.7
40	Graph sketching	27.7
42a	Defining Δy	21.0
42c	Defining $\frac{dy}{dx}$	33.0
43	Differentiation	15.3
44	Integrating to find a volume	6.6
45	Logarithms used in calculations	1.7
46	Geometric Progressions	21.3
48	Annuities-accumulated value	32.7
49	Permutations-Combinations	1.0
50	Binomial Theorem-Approximations	16.7
51	Maximum value of a function	4.0
52	Integrating to find areas	1.3
53	Empirical Formulae	11.7

There were only two questions in this list from the first part of the paper. Of the 19 questions in the second part of the paper, 16 appear in this list.

The criteria adopted here has been arbitrary. From an inspection of these four tables it is evident that pupils found the questions on the second half of the paper difficult or extremely difficult. From this it may be inferred that the problem solving ability of students of this group is strictly limited.

The following table shows the % of candidates who were able to secure 50% or more of marks awarded for any one problem. For obvious reasons, questions 1 - 34 are not included here.

TABLE CLIII

% of Candidates Who Were Able to Secure 50% or More of Marks.

Question	Marks	Topic	% of Candidates
35	4	Ratio Inequalities	2.3
36	4	Joint Variation	48.7
37	6	Graph Sketching	57.7
38	3	Graph Sketching	47.0
39	6	Maxima-Minima	3.3
40	5	Graph sketching	38.0
41	4	Logarithms	36.7
42	5	Definitions	27.0
43 1.	5	Differentiation	24.7
44	6	Integration (volumes)	22.0
45 1.	5	Solution of Equations	12.3
46 2.	3	Geometric Progressions	34.3
47 2.	3	Annuities (No interest)	40.7
48	4	Annuities-(Interest bearing)	39.7
49 2.	3	Permutations-Combinations	1.0
50	4	Binomial Theorem	19.7
51 1.	5	Maximum Values	34.9
52 1.	5	Integration (Areas)	7.0
53 1.	5	Empirical Formulae	34.4

1. Half marks counted as 3 to 5 inclusive.
2. Half marks counted as 2 to 3 inclusive.

42c not included.

From a perusal of the examination papers of 1933, 1935, 1938, 1940, 1941, 1942, 1944 and 1948 there is no doubt whatsoever in the writers mind as to the relative difficulty of the 1948 paper. In a comparison, on the basis of whether or not the questions were taken directly from the text book, the 1948 paper was found to be the easiest of the 8 papers compared. The test is certainly a valid one as can be seen from Fig. I p.3, for it consistently placed students into different categories. From an examination of Tables CXLIX, CL, CLI, CLII, CLIII; Chapter X there is no doubt whatsoever that the group under consideration has been placed in its proper category. From the results as shown in Table CLIII, the writer holds the opinion that this category is too high and that these students should be in Category "C" i.e. holding a pass entitling them to a High School Diploma but not eligible for University Entrance.

II Was the Examination a Fair Test

The relative difficulty of the paper can be judged from the following table. This table shows the number of questions which candidates attempted. N represents the number of candidates who attempted the number of questions as given in the column "No. of questions attempted".

TABLE CLIV

Number of Questions Attempted [1].

No. of questions Attempted	N	%
53	62	20.7
52	50	16.7
51	59	19.7
50	36	12.0
49	24	8.0
48	19	6.3
47	13	4.3
46	9	3.0
45	9	3.0
44	3	1.0
43	9	3.0
42	1	0.3
41	4	1.3
40	2	0.7
	300	100.0

1. See Chapter II

Thus, on the average 50.7 questions were attempted by each candidate. Therefore, on the basis of this table the examination was a fair test.

III How Adequately Has the Examination Covered the Course?

From an examination of Chapter VII, VIII and IX there is no doubt that the Examination covered the course quite thoroughly. Some parts which were omitted are cited here:

1. Differential equations.
2. Growth functions.

IV What does a Pupil at the Lowest Matriculation Level Know?

Chapter VII, VIII, and IX contain this information, and it will not be repeated here.

V The Standard

An examination of Table II, Chapter I reveals that 60% of the candidates received a "B" standing or better i.e. 50% or better while 40% received a "C" standing or less i.e. less than 50%. In other words 60% of those who wrote Grade XII examinations were eligible to enter University. On the basis that a "C" standing qualifies a student for a

"High School Diploma", 85.5% of the candidates passed the examination in Algebra II. This leaves 14.5% of the candidates who failed completely.

An examination of literature dealing with "Intelligence and School Success" and Intelligence and College Success, reveals the fact that on the average 31% of the population has an I.Q. of 110 or more.

Gates and Jersild in "Educational Psychology" have advanced the figure of an I. Q. of 120 for success in college i.e. "It is probable that an I.Q. of at least 120 is needed to do acceptable work in a first class college with an average expenditure of time and energy." If we set the I.Q. necessary for college success at 110, we note that only 31% of the population is able to succeed in college. It is true that pupils have been selected to some extent before they reach Grade XII but on the whole a greater portion of the youth of today is going to school than ever before. Gates and Jersild give a figure of 100 I.Q. as necessary for success in High School. On the average 53% of the population has

an I.Q. of 100 or more. Thus on the basis of these figures, the figure of 60% passes, i.e. eligibility for University entrance in 1948 is too high. It is true that we assume that those just at the Matriculation level will not attempt University but why should we give them this certificate and place them on the same level as an Honors Student.

Because of the method by means of which we allow this great number of students to pass, there is no doubt that we are allowing our standards to drop. An examination of the results of Chapter IX and X will bear this out.

An examination of the following two tables will further bear out the fact that the Standards required for a pass are low. In conjunction with these, it must be borne in mind that on the average a student attempted 50.7 questions.

TABLE CLV

Frequency of Scores Questions 1 to 34.

Score Range	N	%
64 - 60	1	0.3
59 - 55	3	1.0
54 - 50	39	13.0
49 - 45	93	31.0
44 - 40	118	39.3
39 - 35	36	12.0
34 - 30	10	3.3

The total mark on this section of the paper was 68. The average score $\bar{X}$ = 44.6.

TABLE CLVI

Frequency of Scores Question 35 - 53.

Score Range	N	%
43 - 40	4	1.3
39 - 36	6	2.0
35 - 32	39	13.0
31 - 28	77	25.7
27 - 24	98	32.7
23 - 20	53	17.7
19 - 16	17	5.7
15 - 12	5	1.6
11 - 8	1	0.3

The total mark on this section of the paper was 85. The average Score $\overline{X}$ = 27.1

VI General Conclusions and Comparison with General Conclusions of Former Studies.

From a perusal of Tables CXLIX, CL, CLI, and CLII, Chapter X it appears that there are outstanding deficiencies in the ability of students in the following:

1. The limiting process.
2. Logarithms and Logarithmic manipulation.
3. Solution of problems involving ratio inequalities.
4. Graph sketching.
5. Difficult integration.
6. Difficult differentiation.
7. Permutations and Combinations.
8. Accumulated value of annuities.
9. Geometric Progressions.
10. Approximations using the Binomial Theorem.
11. Maxima-Minima.
12. Empirical Formulae.

I. In a general comparison with Dr. Lazerte's conclusions in 1935, the deficiencies of students then seems to be the same as the deficiencies of students in 1948.

. II. The conclusions reached by Mr. Miller in 1940-1941 and 1942 are similar to the conclusions which can be drawn from the 1948 examination.

(1) The examination was excellent in allotment of marks for each section of the text. (The 1948 examination followed the trend of past examinations in this regard).

(2) The problems of the examination seem to be fair samples of those in the text in regard to difficulty.

(3) Some problems were relatively easy and some problems were relatively difficult.

The following table shows the results from the standpoint of % of correct responses on any one topic in 1940-1941-1942 and in 1948.

TABLE CLVII

% of Correct Responses for any one Topic 1940-1941-1942 and 1948.

Topic	% of Correct Responses 1940-41-42	1948
Ratio Proportion & Variation	36	58
Functions in one variable	37	55
Limits and Gradients	38	50.3
Differentiation	41	37.7
Integration	40	23.0
Sequence and Series	28	56.4
Permutations & Combinations	31	28.5
Binomial Theorem	33	41.3
Empirical Formulae	20	41.1

A comparison between results is not valid because of differences in selection of population, different types of examinations etc.

The most outstanding weakness found in 1940-1941 and 1942 was in simple Arithmetic. This is a parallel situation with the 1948 results.

Thus, broadly speaking results of former studies, although no strict comparison can be made, seem to be similar to the finding in 1948. Again, mention must be made of the difference in examinations and the manner in which the cases under consideration were selected.

VII Marking

Greater care should be exercised by the persons who are marking papers to assure that candidates are given full marks for treatment of problems. Instances were found (see Negative Analysis Questions 35-53) where candidates were not given sufficient marks. Markers are too prone in many cases to award marks for work in the column of the Examination booklet headed "Rough Work Only (No marks for works in this space)".

The following example of an actual case illustrates what can happen when improper attention is paid to marking.

This candidate, whose raw score was 70, received a final % grading of 51. The candidate attempted Problem 49 for which 3 marks were given. The solution was incorrect and yet 8 marks were awarded for the question. Thus this candidate received a bonus of 8 marks. In reality then, his raw score should have been 62 which corresponds to a % grading of 44.

VIII Recommendations

1. The investigation has revealed that students in this category have a weak mathematical background. The great number of mechanical errors indicates that there should be an insistence on the part of teachers of Mathematics upon accuracy of details. This should not be left till Grade XII but should begin with the students first days in school.

2. The text book which is being used should be revised so as to include more practise exercises on fundamentals of the course. Mention has already been made of the lack of exercises on integration.

3. The weakness of students in logarithmic manipulation and calculation shows that this topic should be dealt with in Grade XII. Teachers of Mathematics should insist on accuracy in calculation by logarithms.

4. The basic ideas of significant figures should receive some attention in Grade XII.

5. The disparity between the results of Part I of the examination i.e. the multiple choice part of the examination seems to suggest that the Multiple choice type of question might be replaced by the short one step type of question which was used prior to 1946.

6. Following are some suggestions for possible future studies:

(a) To find the correlation between candidates I.Q.'s and Algebra scores.

(b) Setting of a final Algebra 2 examination similar to that of 1935 for example and then comparing results with the 1935 study.

7. The investigation has revealed that candidates are being allowed a "B" standing in Algebra 2 with a very limited knowledge of Mathematics and the Algebra 2 Course. As a "B" standing allows these students to continue with higher academic training, there is no doubt that if we allowed these students to enter University, the standards of the University will be perceptibly lowered in time. Many suggestions have been offered as to how this situation could be remedied i.e. to keep University standards high. Following are some suggestions:

(1) That the university consider applications for admittance from "H" and "A" graduates only.

(2) That two examinations be given in Algebra 2, for example - one for those who wished to continue on to University, the other for those who wish a High School Diploma only.

(3) That each University faculty set up a committee to interview prospective students interested in studying in the faculty. The writer favors this method and feels that the suitability of a candidate cannot be gauged by an impersonal figure

on a transcript. Then too the committee, acting as guidance officers could steer the prospective candidate, if he were not suitable, into a field where he could succeed.

BIBLIOGRAPHY

Gates and Jersild - Education Psychology-New York. The MacMillan Company 1942.

Hardy R. H. - Pure Mathematics-Cambridge University Press 1946.

Minnick J. H. - Teaching Mathematics in the Secondary Schools-New York Prentice Hall 1939.

Remmers and Gage - Educational Measurement and Evaluation-New York, Harper and Brothers.

Pintner - Educational Testing-New York Henry Holt and Company.

- Proceeding of the First Canadian Mathematical Congress, Montreal 1945. University of Toronto Press 1946.

LaZerte, M. E. - Summaries of Investigations in the Mathematical Field. A report to the Carnegie Corporation.

Hooper, S. W. - A Report on the Algebra 3 Papers of June 1938. An unpublished M.A. Thesis available in the Library of the University of Alberta.

Miller, H. E. - A Report on the Algebra 2 Papers of 1940-1941-1942. An unpublished M.Ed. Thesis available in the Library of the University of Alberta.

Miller, V. I. - Student Attainment in the Algebra 2 Examinations of 1944. An unpublished M.Ed. Thesis available in the Library of the University of Alberta.

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